Sufficient Conditions for Generic Simultaneous Pole Assignment and Stabilization of Linear MIMO Dynamical Systems

B. K. Ghosh and X. A. Wang

***Abstract*—It has been shown that a generic r-tuple of m input p output dynamical systems is simultaneously stabilizable (pole assignable) if $r < m + p$ and the McMillian degrees of the systems are not too different. This result improves the currently known upper bound: $r \leq \max(m, p)$ on the number of plants. In the case $r \leq \max(m, p)$, an upper bound on the degree of the simultaneous pole assigning compensator has also been derived, which improves the bound obtained by Ghosh and Byrnes in 1983.**

***Index Terms*—Dependent compensator, dynamic compensator, generic, pole assignment, simultaneous stabilization.**

I. Introduction

The problem of simultaneous stabilization consists of answering the following question.

Given an r-tuple $G_1(s), \cdots, G_r(s)$ of $p \times m$ proper transfer functions, does there exist a compensator $K(s)$ such that the closed-loop systems $G_i(s)(I+K(s)G_i(s))^{-1}$, $i = 1, \cdots, r$ are internally stable?

Likewise the problem of simultaneous pole assignment consists of finding a compensator $K(s)$ of degree q, if possible, such that the closed-loop characteristic polynomials of each of the systems $G_i(s)(I + K(s)G_i(s))^{-1}$, $i = 1, \cdots, r$ of degree n_i, $i = 1, \cdots, r$, respectively, are precisely a set of r *a priori* chosen polynomials of degree $n_i + q$, $i = 1, \cdots, r$, respectively.

The problems of generic simultaneous stabilization and pole assignment was originally considered by Saeks and Murray [10] for a pair of single-input/single-output plants. For a suitable topology on the space of $p \times m$ proper transfer functions we refer to Hazewinkel and Kalman [16], Clark [14], Byrnes and Hurt [15]. For a topology on the r-tuple of plants, we consider the product topology. In 1980, Saeks and Murray [10] show that a generic pair of single-input/single-output plants is not simultaneously stablizable (hence not simultaneously pole assignable) by a dynamic compensator. Motivated by the negative result of [10], Vidyasagar and Viswanadham [11] considered the problem of simultaneous stabilization for the case when $\max(m, p) > 1$ and showed (in 1981) that a generic pair of $p \times m$ plants is simultaneously stablizable if $\max(m, p) > 1$. Both [10] and [11] were published in the year 1982, at which point it was unclear if a generic r-tuple of plants would be simultaneously stablizable if $r > 2$. In 1983, Ghosh and Byrnes [4] showed that r could be chosen as large as $\max(m, p)$. In fact, it was shown in [4] that a generic r-tuple of $p \times m$ plants is simultaneously pole assignable (hence stablizable) if $r \leq \max(m, p)$. Furthermore, if $\min(m, p) = 1$, it was shown [4] that $r \leq \max(m, p)$ is necessary and sufficient for generic simultaneous pole assignment and generic simultaneous stabilization. An open question since 1983 has been to ascertain if a generic r-tuple of $p \times m$ plants is simultaneously stablizable or pole assignable when $\min(m, p) > 1$ and when $r > \max(m, p)$. Of course it is a trivial dimension counting argument that a generic r-tuple of plants is not simultaneously pole assignable if $r \geq m + p$. In this paper, we would therefore ask the following question.

Question 1.1: Is it true that a generic r-tuple of $p \times m$ plants is simultaneously pole assignable if

$$\max(m, p) < r < m + p?$$

One of the main results (Theorem 4.2) of this paper partially answers Question 1.1, i.e., the answer to Question 1.1 is "affirmative" if there are at least $\min(m, p)$ systems whose McMillian degrees are not "too different" (please refer to Theorem 4.2 for precise condition). In particular, the answer to Question 1.1 is "affirmative" if $\min(m, p)$ plants have the same degree. We also derive an upper bound on the degree of the simultaneous pole assigning compensator in Theorem 4.2.

We also give a new sufficient condition

$$q + \left(\left\lfloor \frac{q}{\min(m, p)} \right\rfloor + 1\right)(\max(m, p) - r) \geq \sum_{i=1}^{r} \left\lfloor \frac{n_i}{\min(m, p)} \right\rfloor \tag{1.1}$$

for the case $r \leq \max(m, p)$ (Theorem 4.1), where $\lfloor x \rfloor$ is the largest integer less than or equal to x. Note in particular that when $\min(m, p) = 1$, (1.1) reduces to

$$q(\max(m, p) + 1 - r) + \max(m, p) - r \geq \sum_{i=1}^{r} n_i \tag{1.2}$$

which is precisely the inequality obtained in [4]. On the other hand when $r = \max(m, p)$, the smallest degree of the compensator which simultaneously pole assigns $\max(m, p)$ plants generically is given by

$$\sum_{i=1}^{r} \left\lfloor \frac{n_i}{\min(m, p)} \right\rfloor$$

which should be compared with the smallest degree $\sum_{i=1}^{r} n_i$ obtained in [4]. Thus our result improves the result derived by Ghosh and Byrnes [6] in the case when $r \leq \max(m, p)$.

II. Simultaneous Pole Assignment Map

For convenience we propose to use the behavioral framework of linear systems in this paper. Two kinds of representations for an m-input, p-output system are used.

Kernel Representation (which is called autoregressive representation in [12])

$$P\left(\frac{d}{dt}\right) w(t) = 0$$

where $P(s)$ is a $p \times (m + p)$ full rank polynomial matrix.

Image Representation (which is called moving average representation in [13])

$$w(t) = Q\left(\frac{d}{dt}\right) v(t)$$

where $Q(s)$ is a $(m + p) \times m$ full rank polynomial matrix.

The McMillan degree of a system in either representation is the maximal degree of the full size minors of the polynomial matrix. The row degrees of $P(s)$ [respectively, column degrees of $Q(s)$] are the highest degree of the entries on each rows of $P(s)$ [respectively, columns of $Q(s)$]. A representation is called minimal if the full size minors of

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the polynomial matrix are relative prime and sum of the row degrees (column degrees) is equal to its McMillan degree.

The relation between the kernel and image representations is the following: If a $p \times (m+p)$ matrix $P(s)$ is minimal, there exists [3] a unique (up to column equivalence) $(m+p) \times m$ minimal polynomial matrix $Q(s)$ such that

$$P(s)Q(s) = 0.$$

Then $w(t) = Q(d/dt)v(t)$ is the image representation of the system $P(d/dt)w(t) = 0$, where

$$v(t) = P_1\left(\frac{d}{dt}\right)w(t)$$

for any $m \times (m+p)$ polynomial matrix $P_1(s)$ such that

$$\begin{bmatrix} P(s) \\ P_1(s) \end{bmatrix}$$

is unimodular.

If an input–output system is given in frequency domain by

$$\hat{y}(s) = G(s)\hat{u}(s)$$

let

$$G(s) = D_l^{-1}N_l(s) = N_r(s)D_r^{-1}(s)$$

be left and right coprime factorizations of the transfer function, then kernel and image representations of the system are given, respectively, by

$$\left[N_l\left(\frac{d}{dt}\right), -D_l\left(\frac{d}{dt}\right)\right]\begin{bmatrix} u(t) \\ y(t) \end{bmatrix} = 0$$

and

$$\begin{bmatrix} u(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} D_r\left(\frac{d}{dt}\right) \\ N_r\left(\frac{d}{dt}\right) \end{bmatrix} v(t)$$

where $\hat{v}(s) = D_r^{-1}(s)\hat{u}(s)$.

Let r systems be given by

$$P_i\left(\frac{d}{dt}\right)w(t) = 0, \qquad i = 1, \cdots, r$$

where each $P_i(s)$ is a $p \times (m+p)$ full rank polynomial matrix of McMillan degree n_i, and let the compensator be given by

$$w(t) = Q\left(\frac{d}{dt}\right)v(t)$$

for an $(m+p) \times p$ full rank polynomial matrix $Q(s)$ of McMillan degree q. Then the closed-loop systems become

$$P_i\left(\frac{d}{dt}\right)Q\left(\frac{d}{dt}\right)v(t) = 0, \qquad i = 1, \cdots, r.$$

If $\det P_i(s)Q(s)$ is a nonzero polynomial, then it is the closed loop characteristic polynomial of the system.

Let $\mathcal{M}_q$ be the set of all $(m+p) \times m$ polynomial matrices whose sum of the column degrees is less than or equal to q. Then every dynamic compensator of degree at most q has an image representation in $\mathcal{M}_q$.

Definition 2.1: For r-tuple systems $P_1(s), \cdots, P_r(s)$, the simultaneous pole assignment map

$$\chi: \mathcal{M}_q \to \mathbb{R}^{n_1 + \cdots + n_r + rq + r}$$

is defined by

$$\begin{aligned} \chi(Q) &= (\chi_1(Q), \cdots, \chi_r(Q)) \\ &= (\det P_1(s)Q(s), \cdots, \det P_r(s)Q(s)) \end{aligned} \tag{2.1}$$

where a polynomial $a_0 + a_1 s + \cdots + a_k s^k$ is identified with a point $(a_0, a_1, \cdots, a_k) \in \mathbb{R}^{k+1}$.

Since χ is homogeneous in the coefficients of Q, χ is onto if we can show that a small open ball of the origin is contained in $\chi(\mathcal{M}_q)$.

Definition 2.2: A compensator $Q(s)$ is called a simultaneous dependent compensator of $P_1(s), \cdots, P_r(s)$ if

$$\det P_i(s)Q(s) \equiv 0, \qquad i = 1, \cdots, r.$$

Theorem 2.3: The simultaneous pole assignment map χ is onto if there is a simultaneous dependent compensator $Q(s) \in \mathcal{M}_q$ such that the Jacobian

$$d\chi_Q: T_Q \to \mathbb{R}^{n_1 + \cdots + n_r + rq + r}$$

of χ at $Q(s)$ is onto.

Proof: χ maps a small neighborhood of Q onto a small neighborhood of 0 by the inverse function theorem. Since χ is homogeneous, the whole $\mathbb{R}^{n_1 + \cdots + n_r + rq + r}$ is contained in the image. ■

Theorem 2.4: The Jacobian $d\chi_Q$ is given by

$$d\chi_Q(X(s)) = (\operatorname{tr}(R_1(s)X(s)), \cdots, \operatorname{tr}(R_r(s)X(s)))$$

where

$$R_i(s) = \operatorname{adj}(P_i(s)Q(s))P_i(s).$$

The proof is similar to the proof of [9, Th. 3.10]

III. Existance of Simultaneous Dependent Compensators

In order to use Theorem 2.3, one needs first to find a simultaneous dependent compensator. In this section we give some results concerning the existence of simultaneous dependent compensators and their degrees, and introduce systematic ways to find a simultaneous dependent compensator.

First let us make a simple dimension count. The space of all compensators of degree at most q has dimension $q(m+p) + mp$, and there are $n_1 + \cdots + n_r + rq$ closed-loop poles. So a necessary condition for arbitrary simultaneous pole assignment is

$$q(m+p) + mp \geq n_1 + \cdots + n_r + rq.$$

From this fact we have the following.

Proposition 3.1: If $mp < n_1 + \cdots + n_r$, then a necessary condition for arbitrary simultaneous pole assignment by dynamic compensator is

$$r \leq m + p - 1$$

which is also a necessary condition for the simultaneous pole assignability of the generic r-tuple of systems.

In fact, we are going to show that the condition $r \leq m + p - 1$ also guarantees the existence of a simultaneous dependent compensator.

Theorem 3.2: If $r \leq \max(m, p)$, then a simultaneous dependent compensator of degree at most q exists where q is the smallest integer satisfying

$$\begin{aligned} q + \left(\left\lfloor \frac{q}{\min(m, p)} \right\rfloor + 1\right)(\max(m, p) - r) \\ \geq \sum_{i=1}^{r} \left\lfloor \frac{n_i}{\min(m, p)} \right\rfloor. \end{aligned} \tag{3.1}$$

Proof: Without loss of generality we assume $p \le m$ (otherwise consider image representation of the system and kernel representation of the compensator).

Let $P_i(s)$ be minimal and $\alpha_i(s)$ be the row of $P_i(s)$ with the smallest row degree. We would then have $\deg\,\alpha_i(s) \le \lfloor n_i/p \rfloor$.

Let $\{\beta_1(s), \cdots, \beta_{m+p-r}(s)\}$ be a minimal basis of the dual space (see [3] for definition) of

$$\alpha(s) := [\alpha_1(s)^T, \cdots, \alpha_r(s)^T]^T$$

and arrange the order such that

$$\deg\,\beta_1(s) \le \deg\,\beta_2(s) \le \cdots \le \beta_{m+p-r}(s).$$

Recall that we have the inequality $m+p-r \ge p$. Let us now define $Q(s) = [\beta_1(s), \cdots, \beta_p(s)]$. It follows that $\det\,P_i(s)Q(s) \equiv 0$, $i = 1, \cdots, r$ because each $P_i(s)Q(s)$ has a zero row.

Now we show that the sum of the column degrees of Q is less than or equal to q. We have $\deg\,\beta_1 + \cdots + \deg\,\beta_{m+p-r} \le \deg\,\alpha_1 + \cdots + \deg\,\alpha_r$ and they are equal if and only if the polynomial matrix $\alpha(s)$ is minimal [3]. Utilizing the condition of the q, we have

$$\begin{aligned}\deg\,\beta_1 + \cdots + \deg\,\beta_{m+p-r} &\le \deg\,\alpha_1 + \cdots + \deg\,\alpha_r \\ &\le \left\lfloor \frac{n_1}{p} \right\rfloor + \cdots + \left\lfloor \frac{n_r}{p} \right\rfloor \\ &\le q + \left(\left\lfloor \frac{q}{p} \right\rfloor + 1 \right)(m-r).\end{aligned}$$

If $\deg\,\beta_1 + \cdots + \deg\,\beta_p > q$ then we have $\deg\,\beta_p \ge \lfloor q/p \rfloor + 1$, which implies that $\deg\,\beta_i \ge \lfloor q/p \rfloor + 1$, for all $i > p$, and

$$\begin{aligned}&\deg\,\beta_1 + \cdots + \deg\,\beta_{m+p-r} \\ &\quad = (\deg\,\beta_1 + \cdots + \deg\,\beta_p) \\ &\qquad + (\deg\,\beta_{p+1} + \cdots + \deg\,\beta_{m+p-r}) \\ &\quad > q + \left(\left\lfloor \frac{q}{p} \right\rfloor + 1 \right)(m-r)\end{aligned}$$

a contradiction. ■

As one can see, the proof only uses the condition $r \le m$ not the condition $p \le m$. Therefore we also proved the following result.

Theorem 3.3: If $r \le \min(m, p)$, then a simultaneous dependent compensator of degree at most q exists where q is the smallest integer satisfying one of the inequalities (3.1) and

$$\begin{aligned}q + \left(\left\lfloor \frac{q}{\max(m, p)} \right\rfloor + 1 \right)(\min(m, p) - r) \\ \ge \sum_{i=1}^{r} \left\lfloor \frac{n_i}{\max(m, p)} \right\rfloor.\end{aligned} \tag{3.2}$$

The estimate of q from (3.2) is sometimes smaller than that from (3.1). For example, if $p = 3$, $m = 4$, $r = 2$, $n_1 = n_2 = 7$, the smallest q satisfying (3.2) is 1, but the smallest q satisfying (3.1) is 2.

The proof also gives a systematic way to construct a simultaneous dependent compensator when $r \le \max(m, p)$.

Example 3.4: Consider the 2-input 2-output systems

$$P_1 := \begin{bmatrix} 2s + s^2 - s^3 & 2 + s^3 & s^2 - s^3 & 2s - s^2 + s^3 \\ 1 - s + s^2 & 1 + 2s & -1 - s & 1 + 2s + 2s^2 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} -s + s^2 & -1 + 2s + s^2 & 1 + 2s + 2s^2 & 2 + 2s^2 \\ -1 + 2s & -1 & 2 - s & -1 + 2s \end{bmatrix}.$$

By Theorem 3.2, a simultaneous dependent compensator of degree 3 exists. To find it, we pick a row of the smaller degree from each of the P_1 and P_2

$$\begin{aligned}\alpha_1 &= [1 - s + s^2 \quad 1 + 2s \quad -1 - s \quad 1 + 2s + 2s^2] \\ \alpha_2 &= [-1 + 2s \quad -1 \quad 2 - s \quad -1 + 2s]\end{aligned}$$

and find a minimal basis of the dual space span$(\alpha_1, \alpha_2)^\perp$

$$Q = [\beta_1, \beta_2] = \begin{bmatrix} -6 - 16s & 1478 + 4470s - 1992s^2 \\ 1 - 26s & -5349 - 487s + 590s^2 \\ -16s & -6264s - 1992s^2 \\ 5 + 8s & 3871 - 5813s + 996s^2 \end{bmatrix}.$$

Then it is a simultaneous dependent compensator. One can check that the linear map

$$d\chi_Q(X(s)) = (\mathrm{tr}(\mathrm{adj}(P_1 Q) P_1 X), \mathrm{tr}(\mathrm{adj}(P_2 Q) P_2 X))$$

is onto, where $X(s)$ are polynomial matrices of column degrees at most (1, 2); therefore, the systems are simultaneously pole assignable with a single dynamic compensator of degree 3.

Next we consider the cases of $r \le m + p - 1$.

Theorem 3.5: If $\min(m, p) < r \le m + p - 1$, then a simultaneous dependent compensator of degree at most

$$q = \sum_{i=1}^{\min(m, p)} \left\lceil \frac{n_i + \sum_{j=\min(m,p)+1}^{r} \lfloor n_j / \min(m, p) \rfloor}{m + p - r} \right\rceil$$

exists.

Proof: Without loss of generality, we can assume $p \le m$. Let $P_1(s), \cdots, P_r(s)$ be minimal.

Let $\alpha_i(s)$ be the row of $P_i(s)$ with the smallest degree, $i = p + 1, \cdots, r$. Then $\deg\,\alpha_i(s) \le [n_i/p]$. Let $\beta_i(s)$ be a column vector of the smallest degree of the dual space of

$$[P_i(s)^T, \alpha_{p+1}(s)^T, \cdots, \alpha_r(s)^T]^T$$

for $i = 1, \cdots, p$. Then

$$\deg\,\beta_i(s) \le \left\lceil \frac{n_i + \sum_{j=\min(m,p)+1}^{r} \lfloor n_j / \min(m, p) \rfloor}{m + p - r} \right\rceil.$$

Let $Q = [\beta_1(s), \cdots, \beta_p(s)]$ (replace any linearly dependent vector with arbitrary vector in dual space of $\mathrm{span}\{\alpha_{p+1}, \cdots, \alpha_r\}$). Then

$$\deg\,Q(s) \le \sum_{i=1}^{\min(m, p)} \left\lceil \frac{n_i + \sum_{j=\min(m,p)+1}^{r} \lfloor n_j / \min(m, p) \rfloor}{m + p - r} \right\rceil$$

and $Q(s)$ is a simultaneous dependent compensator. ■

Example 3.6: Consider 2-input 2-output systems

$$P_1 = \begin{bmatrix} -1 + s & -s & 1 & 2s \\ 2 + s & 1 - s & s & -1 + s \end{bmatrix}$$

$$P_2 = \begin{bmatrix} -1 - s & 1 + s & 2 & 2s \\ -1 - s & -s & 2 - s & 1 - s \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 1 - s - s^2 & 1 + s^2 & -1 - s^2 & -1 + 2s^2 \\ s & 1 + 2s & 2 - s & -s \end{bmatrix}.$$

By Theorem 3.5, a simultaneous dependent compensator of degree 6 exists. Let

$$\alpha_3 = [s \quad 1+2s \quad 2-s \quad -s]$$

be the last row of P_3. Then

$$\left(\text{row span}\begin{bmatrix} -1+s & -s & 1 & 2s \\ 2+s & 1-s & s & -1+s \\ s & 1+2s & 2-s & -s \end{bmatrix}\right)^{\perp} = \text{span}\begin{bmatrix} 1-2s+4s^2+2s^3 \\ -2+14s-s^2-4s^3 \\ 1-5s-8s^2-3s^3 \\ 14s-3s^3 \end{bmatrix}$$

and

$$\left(\text{row span}\begin{bmatrix} -1-s & 1+s & 2 & 2s \\ -1-s & -s & 2-s & 1-s \\ s & 1+2s & 2-s & -s \end{bmatrix}\right)^{\perp} = \text{span}\begin{bmatrix} -3s-15s^2+6s^3 \\ -2+3s+11s^2-4s^3 \\ 1-10s^2-7s^3 \\ -2-6s-s^2+5s^3 \end{bmatrix}.$$

Therefore

$$Q = \begin{bmatrix} 1-2s+4s^2+2s^3 & -3s-15s^2+6s^3 \\ -2+14s-s^2-4s^3 & -2+3s+11s^2-4s^3 \\ 1-5s-8s^2-3s^3 & 1-10s^2-7s^3 \\ 14s-3s^3 & -2-6s-s^2+5s^3 \end{bmatrix}$$

is a simultaneous dependent compensator. For such Q the linear map

$$d\chi_Q(X(s)) = (\text{tr}(\text{adj}(P_1Q)P_1X), \cdots, \text{tr}(\text{adj}(P_3Q)P_3X))$$

is onto, where $X(s)$ are polynomial matrices of column degrees at most (3, 3), and therefore the systems are simultaneously pole assignable by a single compensator of degree 6.

One can easily construct a simultaneously pole assigning compensator using the Newton's method for any r-tuple of systems satisfying the condition of Theorem 2.3 (see [9]).

IV. Simultaneous Pole Assignment for Generic Systems

In this section we give estimates on the degree of the simultaneous pole assigning compensator for a generic r-tuple of systems in the cases $r \leq \max(m, p)$ and $r < m+p$, respectively. It is not too difficult to show that the set of all simultaneous pole assignable r-tuples is Zariski open, so if one can show that it is nonempty, then such r-tuples are generic. The proofs involve construction of one such r-tuple systems, and estimate the degrees of the pole assigning compensator. Because of the restriction on the length of the paper, we omit the proofs here. Interested readers can refer to [8].

Theorem 4.1: If $r \leq \max(m, p)$, then a generic r-tuple of $p \times m$ proper plants of degree $n_i, i = 1, \cdots, r$, respectively, can be pole assigned by a compensator of degree less than or equal to q, where q is the smallest integer satisfying

$$q + \left(\left\lfloor \frac{q}{\min(m, p)} \right\rfloor + 1\right)(\max(m, p) - r) \geq \sum_{i=1}^{r} \left\lfloor \frac{n_i}{\min(m, p)} \right\rfloor. \tag{4.1}$$

Theroem 4.2: If $\max(m, p) < r < m+p$ and (relabel the plants if necessary)

$$\left| \left\lfloor \frac{n_j + N}{m+p-r} \right\rfloor - \left\lfloor \frac{n_l + N}{m+p-r} \right\rfloor \right| \leq 1, \quad 1 \leq j < l \leq \min(m, p) \tag{4.2}$$

where

$$N = \sum_{i=\min(m, p)+1}^{r} \left\lfloor \frac{n_i}{\min(m, p)} \right\rfloor$$

then a generic r-tuple of $p \times m$ proper plants of degree $n_i, i = 1, \cdots, r$, respectively, can be pole assigned by a compensator of degree less than or equal to

$$q = \sum_{i=1}^{\min(m, p)} \left\lfloor \frac{n_i + N}{m+p-r} \right\rfloor.$$

V. Conclusion

To conclude, this paper completely settles an important open question on generic simultaneous stabilization and pole assignment—how many plants can be generically pole assigned and therefore stabilized? It is shown that the number of plants has to be strictly less than $m+p$ provided that the degrees of the plants are *not too different*. In each of the cases when the r-tuple is generically pole assignable, it is pole assignable by a compensator of an *a priori* bounded degree. Estimates of the bound on the degree has been provided in this paper for two separate cases, $r \leq \max(m, p)$ and $r < m+p$ and these estimates improve known results in the literature.

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Stabilization of a Class of Linear Time-Varying Systems via Modeling Error Compensation

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***Abstract*—The aim of this paper is to show that modeling error compensation techniques [1] can be extended for the case of single-input/single-output minimum-phase, linear time-varying systems whose parameters can drift arbitrarily fast.**

***Index Terms*—Modeling error compensation, time-varying parameters, SISO.**

I. Introduction

In a recent work, Sun *et al.* [1] proposed a new robust controller design method for single-input/single-output (SISO) minimum-phase linear systems. The design approach consists of a modeling error compensator (MEC). The central idea is to compensate the error due to uncertainty by determining the modeling error via plant input and output signals and use this information in the design. In addition to a nominal feedback, another feedback loop is introduced using the modeling error and this feedback action is explicitly proportional to the parametric error which is the source of uncertainty.

Most of the work on robust adaptive stabilization for linear time-varying (LTV) systems has been focused in plants with small-in-the-mean parameter variations [2]–[4]. Moreover, in most cases bounded-input/bounded-state stability results are obtained. Thus, the need arises of finding a better alternative for controlling plants with fast time-varying parameters [5].

In this paper we show that the MEC technique proposed by Sun *et al.* [1] can be extended to the case of linear time-varying (LTV) systems. The new element in this paper is an idea to achieve robust control via high-gain observer alone without explicit design of high-gain feedback. Instead of designing a robust state feedback to dominate the uncertain term, the uncertain term is viewed as an extra state that is estimated using a high-gain observer. The estimation of the uncertain term gives the control system some degree of adaptivity. The proposed MEC controller has several advantages over traditional adaptive schemes. The advantages include linearity of control law and straightforward stability proof via Lyapunov functions.

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II. Description of the System

Consider the continuous-time SISO time-varying system modeled as follows:

$$A(s, t)[y(t)] = b(t)u(t) \tag{1}$$

where $A(s, t)$ is a linear time-varying (LTV), polynomial differential operator (PDO) given by $A(s, t) = s^n + a_n(t)s^{n-1} + \cdots + a_1(t)$ and the symbol s denotes the differential operator d/dt. With respect to the LTV plant, the following assumptions are made [5].

Assumption 1: The sign of the gain $b(t)$ is fixed and known and assumed without losing generality to be positive.

Assumption 2: The index n is constant and exactly known. The parameters of the system, $a_i(t)$, $1 \le i \le n$, and $b(t)$, are continuously differentiable. The parameters may not be known and may drift arbitrarily fast, but the values of the parameters and their first time-derivatives are bounded for all $t \ge 0$

$$\begin{aligned} &a_i^{\min} \le a_i(t) \le a_i^{\max}, \qquad |da_i(t)/dt| \le \alpha_i, \qquad 1 \le i \le n \\ &0 < b^{\min} \le b(t) \le b^{\max}, \quad |db(t)/dt| \le \beta. \end{aligned} \tag{2}$$

Suppose that all we know about $b(t)$ is an upper bound $b^{\max}$.

The control objective is to find a continuous-feedback controller to guarantee closed-loop asymptotic stability, while the controller requires only input and output measurements.

It must be pointed out that, for the sake of simplicity in exposition, we have restricted ourselves to the class of systems described by (1). By adding a series of integrators at the input channel, it is not hard to extend the results in this work to the class of minimum-phase systems considered in [5]

$$A(s, t)[y(t)] = B(s, t)[u(t)]$$

where the PDO $B(s, t) = b_{m+1}(t)s^m + b_m(t)s^{m-1} + \cdots + b_1(t)$, $m \le n - 1$ is exponentially stable with rate no longer than $-\gamma_B$ for some $\gamma_B > 0$ (see [5]), and the index m is constant and exactly known.

III. Main Results

To achieve our control objective in the presence of parameter variations, we propose an MEC controller structure [1]. A nominal design is used as a primary design. An additional feedback loop is introduced to improve the robustness of the nominal design.

Let $y^{(i)} = d^i y/dt^i$. By taking $x_i = y^{(i-1)}$, $1 \le i \le n$, we can represent the plant (1) as

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \qquad 1 \le i \le n-1 \\ \dot{x}_n &= -\sum_{k=1}^{n} a_k(t)x_k + b(t)u \\ y &= x_1. \end{aligned} \tag{3}$$

The new element in this paper is an idea to achieve the robust control via a high-gain observer alone without an explicit design of high-gain feedback. Instead of designing a robust state feedback to dominate this uncertain term, we will view the uncertain term as an extra state which is estimated using a high-gain observer.