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Some new results on the simultaneous stabilizability of a family of single input, single output systems

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Abstract: In this paper, we obtain sufficiency conditions on the problem of simultaneous stabilization of a parameterized family of single input, single output plants by a non-switching compensator. The sufficiency conditions include simultaneous stabilization and simultaneous pole assignment of a finite number of plants.

Keywords: Simultaneous stabilization, Simultaneous pole placement, Parameterized family.

1. Introduction

The simultaneous stabilization of a family of systems by a non-switching compensator is described in the continuous time as follows:

Problem 1. Given a family of $p \times m$ linear dynamical systems $G_\lambda(s)$, $\lambda \in \Lambda$, of McMillan degrees n, where Λ is a parameter set, does there exist a compensator $K(s)$ of McMillan degree q such that the closed loop systems $G_\lambda(s)[I + K(s)G_\lambda(s)]^{-1}$ are stable, i.e. have poles in the open left half of the complex plane, for all $\lambda \in \Lambda$?

The above problem is important in the design of a robust compensator which stabilizes a family of plants under gross and local variation in the plant parameters.

In [4], Birdwell, Castanon and Athans introduced the above problem when Λ is finite. In [14,15] Tannenbaum considered the problem of simultaneously stabilizing a family of single input, single output systems $G_\lambda(s) = \lambda g(s)$ where $g(s)$ is a fixed single input, single output system and $\lambda \in [a, b]$ for some $a, b \in \mathbb{R}$ by a stable dynamic compensator. For some recent works on the above problem we refer to [9,12,16,10].

In this paper, we assume $m = p = 1$ and consider the following two families of plants:

$$\mathscr{F}_1 \triangleq \left\{ g_\lambda(s) \colon g_\lambda(s) = \sum_{i=0}^{n-1} [(1-\lambda)\alpha_i + \lambda\beta_i] s^i \Big/ \sum_{i=0}^{n-1} [(1-\lambda)\gamma_i + \lambda\delta_i] s^i + s^n, \right.$$

$$\left. \alpha_i,\ \beta_i,\ \gamma_i,\ \delta_i \in \mathbb{R},\ \lambda \in [0, 1],\ \deg g_\lambda(s) = n\ \forall\lambda \right\}, \tag{1.1}$$

$$\mathscr{F}_2 \triangleq \left\{ g(s) \colon g(s) = \sum_{i=0}^{n-1} a_i s^i \Big/ \left[\sum_{i=0}^{n-1} b_i s^i + s^n \right], \right.$$

$$\left. a_i \in [\alpha_i, \beta_i],\ b_i \in [\gamma_i, \delta_i],\ \alpha_i \le \beta_i,\ \gamma_i \le \delta_i,\ i = 0, \ldots, n-1,\ \deg g(s) = n \right\}. \tag{1.2}$$

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We remark that $\mathscr{F}_1$ is a one-parameter family of plants and represents a gross variation in the plant parameter. The family $\mathscr{F}_2$, on the other hand, represents a local variation around a fixed point in a neighbourhood.

The main result of this paper is to obtain a sufficient condition on the problem of simultaneous stabilization of the family $\mathscr{F}_1$ and the family $\mathscr{F}_2$ by a dynamic compensator

$$k(s)=\left[\sum_{i=0}^{g} c_i s^i\right] \Big/ \left[\sum_{i=0}^{q-1} d_i s^i + s^q\right] \tag{1.3}$$

of McMillan degree q. In order to describe the results, we need the following notation. Let us denote the columns of the $(2q+2)\times(n+q)$ matrix

$$\begin{bmatrix} \alpha_0 & \alpha_1 & \cdots & \cdots & \alpha_{n-1} & 0 & & & 0 \\ \gamma_0 & \gamma_1 & \cdots & \cdots & \gamma_{n-1} & 1 & & & \\ & \alpha_0 & \alpha_1 & \cdots & \alpha_{n-2} & \alpha_{n-1} & 0 & & \\ & \gamma_0 & \gamma_1 & \cdots & \gamma_{n-2} & \gamma_{n-1} & 1 & & \\ & & \ddots & & & & & \ddots & \\ & & & \alpha_0 & \alpha_1 & \cdots & \cdots & \alpha_{n-1} & 0 \\ & & & \gamma_0 & \gamma_1 & \cdots & \cdots & \gamma_{n-1} & 1 \\ & & & & \alpha_0 & \cdots & \cdots & \alpha_{n-2} & \alpha_{n-1} \\ 0 & & & & \gamma_0 & \cdots & \cdots & \gamma_{n-2} & \gamma_{n-1} \end{bmatrix} \tag{1.4}$$

by $\nu_0^{\mathrm{T}},\ldots,\nu_{n+q-1}^{\mathrm{T}}$ respectively, and the columns of matrix (1.4) with α_i replaced by β_i and γ_i replaced by δ_i for $i=0,\ldots,n-1$ by $u_0^{\mathrm{T}},\ldots,u_{n+q-1}^{\mathrm{T}}$ respectively. Furthermore let us denote

$$\begin{bmatrix} c_0 & d_0 & \cdots & c_{q-1} & d_{q-1} & c_q & 1 \end{bmatrix} \triangleq \Psi, \tag{1.5}$$

$$\begin{bmatrix} 1 & s & s^2 & \cdots & s^{n+q-1} \end{bmatrix}^{\mathrm{T}} \triangleq \boldsymbol{s}^{\mathrm{T}} \tag{1.6}$$

We now state the following:

Theorem 1.1. *A sufficient condition that every plant in $\mathscr{F}_1$ is simultaneously stabilizable by a compensator $k(s)$ of degree q is given by the stability of the following four polynomials:*

$$\Psi\left[\nu_0^{\mathrm{T}}, \nu_1^{\mathrm{T}},\ldots,\nu_{n+q-1}^{\mathrm{T}}\right]\boldsymbol{s}^{\mathrm{T}}+s^{n+q}, \tag{1.7}$$

$$\Psi\left[u_0^{\mathrm{T}}, u_1^{\mathrm{T}},\ldots,u_{n+q-1}^{\mathrm{T}}\right]\boldsymbol{s}^{\mathrm{T}}+s^{n+q}, \tag{1.8}$$

$$\Psi\left[\nu_0^{\mathrm{T}}, u_1^{\mathrm{T}}, \nu_2^{\mathrm{T}}, u_3^{\mathrm{T}},\ldots,\left(\nu_{n+q-1}^{\mathrm{T}} \text{ or } u_{n+q-1}^{\mathrm{T}}\right)\right]\boldsymbol{s}^{\mathrm{T}}+s^{n+q}, \tag{1.9}$$

$$\Psi\left[u_0^{\mathrm{T}}, \nu_1^{\mathrm{T}}, u_2^{\mathrm{T}}, \nu_3^{\mathrm{T}},\ldots,\left(u_{n+q-1}^{\mathrm{T}} \text{ or } \nu_{n+q-1}^{\mathrm{T}}\right)\right]\boldsymbol{s}^{\mathrm{T}}+s^{n+q}. \tag{1.10}$$

Remark. In (1.9) and (1.10), the co-efficients of s^{n+q-1} are to be chosen depending upon whether $n+q-1$ is even or odd.

The following corollary of Theorem 1.1 is immediate.

Corollary 1.2. *A sufficient condition that every plant in $\mathscr{F}_1$ is simultaneously stabilizable by a constant gain feedback is given by the simultaneous stabilizability of the following four plants by a constant gain feedback:*

$$\sum_{i=0}^{n-1} \alpha_i s^i \Big/ \left[\sum_{i=0}^{n-1} \gamma_i s^i + s^n\right], \tag{1.11}$$

We remark that $\mathscr{F}_1$ is a one-parameter family of plants and represents a gross variation in the plant parameter. The family $\mathscr{F}_2$, on the other hand, represents a local variation around a fixed point in a neighbourhood.

The main result of this paper is to obtain a sufficient condition on the problem of simultaneous stabilization of the family $\mathscr{F}_1$ and the family $\mathscr{F}_2$ by a dynamic compensator

$$k(s)=\left[\sum_{i=0}^{g} c_i s^i\right] \Big/ \left[\sum_{i=0}^{q-1} d_i s^i + s^q\right] \tag{1.3}$$

of McMillan degree q. In order to describe the results, we need the following notation. Let us denote the columns of the $(2q+2)\times(n+q)$ matrix

$$\begin{bmatrix} \alpha_0 & \alpha_1 & \cdots & \cdots & \alpha_{n-1} & 0 & & & & 0 \\ \gamma_0 & \gamma_1 & \cdots & \cdots & \gamma_{n-1} & 1 & & & & \\ & \alpha_0 & \alpha_1 & \cdots & \alpha_{n-2} & \alpha_{n-1} & 0 & & & \\ & \gamma_0 & \gamma_1 & \cdots & \gamma_{n-2} & \gamma_{n-1} & 1 & & & \\ & & \ddots & & & & & \ddots & & \\ & & & \alpha_0 & \alpha_1 & \cdots & \cdots & \alpha_{n-1} & 0 & \\ & & & \gamma_0 & \gamma_1 & \cdots & \cdots & \gamma_{n-1} & 1 & \\ & & & & \alpha_0 & \cdots & \cdots & \alpha_{n-2} & \alpha_{n-1} & \\ 0 & & & & \gamma_0 & \cdots & \cdots & \gamma_{n-2} & \gamma_{n-1} & \end{bmatrix} \tag{1.4}$$

by $\nu_0^{\mathrm{T}},\ldots,\nu_{n+q-1}^{\mathrm{T}}$ respectively, and the columns of matrix (1.4) with α_i replaced by β_i and γ_i replaced by δ_i for $i=0,\ldots,n-1$ by $u_0^{\mathrm{T}},\ldots,u_{n+q-1}^{\mathrm{T}}$ respectively. Furthermore let us denote

$$\begin{bmatrix} c_0 & d_0 & \cdots & c_{q-1} & d_{q-1} & c_q & 1 \end{bmatrix} \triangleq \boldsymbol{\Psi}, \tag{1.5}$$

$$[1 \quad s \quad s^2 \quad \ldots \quad s^{n+q-1}]^{\mathrm{T}} \triangleq \boldsymbol{s}^{\mathrm{T}} \tag{1.6}$$

We now state the following:

Theorem 1.1. *A sufficient condition that every plant in $\mathscr{F}_1$ is simultaneously stabilizable by a compensator $k(s)$ of degree q is given by the stability of the following four polynomials*:

$$\Psi\left[\nu_0^{\mathrm{T}},\, \nu_1^{\mathrm{T}},\ldots,\nu_{n+q-1}^{\mathrm{T}}\right]\boldsymbol{s}^{\mathrm{T}} + s^{n+q}, \tag{1.7}$$

$$\Psi\left[u_0^{\mathrm{T}},\, u_1^{\mathrm{T}},\ldots,u_{n+q-1}^{\mathrm{T}}\right]\boldsymbol{s}^{\mathrm{T}} + s^{n+q}, \tag{1.8}$$

$$\Psi\left[\nu_0^{\mathrm{T}},\, u_1^{\mathrm{T}},\, \nu_2^{\mathrm{T}},\, u_3^{\mathrm{T}},\ldots,\left(\nu_{n+q-1}^{\mathrm{T}} \text{ or } u_{n+q-1}^{\mathrm{T}}\right)\right]\boldsymbol{s}^{\mathrm{T}} + s^{n+q}, \tag{1.9}$$

$$\Psi\left[u_0^{\mathrm{T}},\, \nu_1^{\mathrm{T}},\, u_2^{\mathrm{T}},\, \nu_3^{\mathrm{T}},\ldots,\left(u_{n+q-1}^{\mathrm{T}} \text{ or } \nu_{n+q-1}^{\mathrm{T}}\right)\right]\boldsymbol{s}^{\mathrm{T}} + s^{n+q}. \tag{1.10}$$

Remark. In (1.9) and (1.10), the co-efficients of s^{n+q-1} are to be chosen depending upon whether $n+q-1$ is even or odd.

The following corollary of Theorem 1.1 is immediate.

Corollary 1.2. *A sufficient condition that every plant in $\mathscr{F}_1$ is simultaneously stabilizable by a constant gain feedback is given by the simultaneous stabilizability of the following four plants by a constant gain feedback*:

$$\sum_{i=0}^{n-1} \alpha_i s^i \Big/ \left[\sum_{i=0}^{n-1} \gamma_i s^i + s^n\right], \tag{1.11}$$

$$\sum_{i=0}^{n-1} \beta_i s^i \Bigg/ \left[\sum_{i=0}^{n-1} \delta_i s^i + s^n \right], \tag{1.12}$$

$$\frac{\alpha_0 + \beta_1 s + \alpha_2 s^2 + \beta_3 s^3 + \cdots + (\alpha_{2i} \text{ or } \beta_{2i+1}) s^{n-1}}{\gamma_0 + \delta_1 s + \gamma_2 s^2 + \delta_3 s^3 + \cdots + (\gamma_{2i} \text{ or } \delta_{2i+1}) s^n}, \tag{1.13}$$

$$\frac{\beta_0 + \alpha_1 s + \beta_2 s^2 + \alpha_3 s^3 + \cdots + (\beta_{2i} \text{ or } \alpha_{2i+1}) s^{n-1}}{\delta_0 + \gamma_1 s + \delta_2 s^2 + \gamma_3 s^3 + \cdots + (\delta_{2i} \text{ or } \gamma_{2i+1}) s^n}. \tag{1.14}$$

Remark. It may be noted that the sufficiency condition of Corollary 1.2 is given by the simultaneous stabilization of finitely many plants studied in [6,13,8,17].

In order to state the next main result, we define the following.

Definition. A compensator $k(s)$ of degree q given by (1.3) is said to be a *positive compensator* if $c_i, d_j \geq 0$, $i = 0, \ldots, q$; $j = 0, \ldots, q-1$.

As an example of a positive compensator, it may be noted that a real time, stable, minimum phase compensator is a positive compensator. In particular, a positive gain is also a positive compensator.
Let us now consider the following theorem.

Theorem 1.3. *A sufficient condition that every plant in $\mathscr{F}_2$ is simultaneously stabilizable by a positive compensator $k(s)$ of degree q is given by the stability of the following four polynomials*:

$$\Psi\left[v_0^{\mathrm{T}}, v_1^{\mathrm{T}}, u_2^{\mathrm{T}}, u_3^{\mathrm{T}}, \ldots\right] \boldsymbol{s}^{\mathrm{T}} + s^{n+q}, \tag{1.15}$$

$$\Psi\left[u_0^{\mathrm{T}}, u_1^{\mathrm{T}}, v_2^{\mathrm{T}}, v_3^{\mathrm{T}}, \ldots\right] \boldsymbol{s}^{\mathrm{T}} + s^{n+q}, \tag{1.16}$$

$$\Psi\left[v_0^{\mathrm{T}}, u_1^{\mathrm{T}}, u_2^{\mathrm{T}}, v_3^{\mathrm{T}}, v_4^{\mathrm{T}}, \ldots\right] \boldsymbol{s}^{\mathrm{T}} + s^{n+q}, \tag{1.17}$$

$$\Psi\left[u_0^{\mathrm{T}}, v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, u_3^{\mathrm{T}}, u_4^{\mathrm{T}}, \ldots\right] \boldsymbol{s}^{\mathrm{T}} + s^{n+q}. \tag{1.18}$$

Remark. It may be noted that the role played by the positivity of the compensator is in ensuring that the coefficients of the closed loop characteristic polynomial lie in the interval $[\psi u_i^{\mathrm{T}}, \psi v_i^{\mathrm{T}}]$, $i = 0, \ldots, n+q-1$, for every choice of the positive vector ψ.

The following corollary of Theorem 1.3 is rather surprising.

Corollary 1.4. *A necessary and sufficient condition that every plant $\mathscr{F}_2$ is simultaneously stabilizable by a gain k is given by the simultaneous stabilizability of the following eight plants by a feedback gain k*:

$$\left[\alpha_0 + \alpha_1 s + \beta_2 s^2 + \beta_3 s^3 + \alpha_4 s^4 + \cdots + (\ ) s^{n-1}\right] / \left[\gamma_0 + \gamma_1 s + \delta_2 s^2 + \delta_3 s^3 + \gamma_4 s^4 + \cdots + (\ ) s^{n-1} + s^n\right], \tag{1.19}$$

$$\left[\alpha_0 + \beta_1 s + \beta_2 s^2 + \alpha_3 s^3 + \alpha_4 s^4 + \cdots + (\ ) s^{n-1}\right] / \left[\gamma_0 + \delta_1 s + \delta_2 s^2 + \gamma_3 s^3 + \gamma_4 s^4 + \cdots + (\ ) s^{n-1} + s^n\right], \tag{1.20}$$

$$\left[\beta_0 + \beta_1 s + \alpha_2 s^2 + \alpha_3 s^3 + \beta_4 s^4 + \cdots + (\ ) s^{n-1}\right] / \left[\delta_0 + \delta_1 s + \gamma_2 s^2 + \gamma_3 s^3 + \delta_4 s^4 + \cdots + (\ ) s^{n-1} + s^n\right], \tag{1.21}$$

$$\left[\beta_0 + \alpha_1 s + \alpha_2 s^2 + \beta_3 s^3 + \beta_4 s^4 + \cdots + (\ ) s^{n-1}\right] / \left[\delta_0 + \gamma_1 s + \gamma_2 s^2 + \delta_3 s^3 + \delta_4 s^4 + \cdots + (\ ) s^{n-1} + s^n\right], \tag{1.22}$$

$$[\alpha_0 + \alpha_1 s + \beta_2 s^2 + \beta_3 s^3 + \alpha_4 s^4 + \cdots + (\ )s^{n-1}]/[\delta_0 + \delta_1 s + \gamma_2 s^2 + \gamma_3 s^3 + \delta_4 s^4 + \cdots + (\ )s^{n-1} + s^n] \quad (1.23)$$

$$[\alpha_0 + \beta_1 s + \beta_2 s^2 + \alpha_3 s^3 + \alpha_4 s^4 + \cdots + (\ )s^{n-1}]/[\delta_0 + \gamma_1 s + \gamma_2 s^2 + \delta_3 s^3 + \delta_4 s^4 + \cdots + (\ )s^{n-1} + s^n] \quad (1.24)$$

$$[\beta_0 + \beta_1 s + \alpha_2 s^2 + \alpha_3 s^3 + \beta_4 s^4 + \cdots + (\ )s^{n-1}]/[\gamma_0 + \gamma_1 s + \delta_2 s^2 + \delta_3 s^3 + \gamma_4 s^4 + \cdots + (\ )s^{n-1} + s^n] \quad (1.25)$$

$$[\beta_0 + \alpha_1 s + \alpha_2 s^2 + \beta_3 s^3 + \beta_4 s^4 + \cdots + (\ )s^{n-1}]/[\gamma_0 + \delta_1 s + \delta_2 s^2 + \gamma_3 s^3 + \gamma_4 s^4 + \cdots + (\ )s^{n-1} + s^n] \quad (1.26)$$

Note. The co-efficient of s^{n-1} depends upon whether or not n is even or odd and is clear from the abov pattern.

Remark. The conditions of Corollary 1.4 can be checked using 'decision algebra' methods and we refer t [6] for details.

Corollary 1.5. *A necessary and sufficient condition that every plant in $\mathscr{F}_2$ is simultaneously stabilizable by a positive gain k is given by the simultaneous stabilizability of the plants* (1.19), (1.20), (1.21) *and* (1.22) *by a positive gain k.*

Corollary 1.6. *A sufficient condition that every plant in $\mathscr{F}_2$ is simultaneously stabilizable by a positiv compensator $k(s)$ of degree q is given by the simultaneous pole assignment of the pair of plants* (1.11), (1.12 *by the positive compensator $k(s)$ of degree q.*

Theorem 1.3 and the associated corollaries may be viewed as a sufficiency condition for the simulta neous stabilization of plants in $\mathscr{F}_2$ by some compensator of degree q. We note that the application o positive compensators in feedback simultaneous stabilization problems is not new. In fact we have show [7] that a triplet of single input, single output plants chosen generically in the topology described in [7] i simultaneously stabilizable by a compensator iff a single input, single output plant is simultaneously partially pole assignable by a stable minimum phase compensator. Moreover we note that a stabl minimum phase compensator is a positive compensator.

2. Proof of Theorem 1.1 and Corollary 1.2

The proof follows from an application of a proposition described by Bose [5] on convex combinations o stable polynomials stated as follows (see also Bialas and Garloff [3]).

Proposition 2.1 (Bose [5]). *Let $p_1(s)$, $p_2(s)$ be two strictly Hurwitz polynomials of degree n with rea co-efficients. Then for any real $\gamma \in [0, 1]$, the polynomial*

$$p_\lambda(s) = (1-\lambda)p_1(s) + \lambda p_2(s) \quad (2.1)$$

is strictly Hurwitz if either

$$m_1(s) = m_2(s) \quad \text{or} \quad n_1(s) = n_2(s) \quad (2.2)$$

where $m_i(s)$ is the even part of $p_i(s)$ and $n_i(s)$ is the odd part of $p_i(s)$, $i = 1, 2$.

We note that a polynomial is strictly Hurwitz if all its roots are in the open left half of the complex plane. The following lemma is an easy extension of Proposition 2.1.

Lemma 2.2. *Assume $p_1(s)$, $p_2(s)$, λ, $p_\lambda(s)$ as in Proposition 2.1. Then for any real $\lambda \in [0, 1]$, the polynomial $p_\lambda(s)$ is stable if the four polynomials $m_1(s)+n_1(s)$, $m_1(s)+n_2(s)$, $m_2(s)+n_1(s)$, $m_2(s)+n_2(s)$ are strictly Hurwitz.*

Proof. By Proposition 2.1 the polynomials $[(1-\lambda)m_1+\lambda m_2]+n_1$ and $[(1-\lambda)m_1+\lambda m_2]+n_2$ are strictly Hurwitz. Hence the polynomial $[(1-\lambda)m_1+\lambda m_2]+[(1-\lambda_1)n_1+\lambda_1 n_2]$ is strictly Hurwitz for λ_1, $\lambda \in [0, 1]$. In particular, for $\lambda_1=\lambda$ the result follows. □

The proof of Theorem 1.1 is now sketched. Let $g_\lambda(s)$ be a plant in $\mathscr{F}_1$ given by (1.1) and let $k(s)$ be a compensator given by (1.3). The closed loop characteristic polynomial is given by

$$\Psi\left\{\left[v_0^{\mathrm{T}}, v_1^{\mathrm{T}}, \ldots, v_{n+q-1}^{\mathrm{T}}\right](1-\lambda)+\left[u_0^{\mathrm{T}}, u_1^{\mathrm{T}}, \ldots, u_{n+q-1}^{\mathrm{T}}\right]\lambda\right\}\boldsymbol{s}^{\mathrm{T}}+s^{n+q}. \tag{2.3}$$

Assuming $p_1(s)$, $p_2(s)$ to be the polynomials given by (1.7) and (1.8) respectively, the proof of Theorem 1.1 now follows from Lemma 2.2. □

The proof of Corollary 1.2 follows from the observation that the polynomials (1.7)–(1.10) are exactly the characteristic polynomials of the plants (1.11)–(1.14) under a constant gain feedback.

3. Proof of Theorem 1.3 and its corollaries

The proof follows from an application of a rather surprising result by Kharitonov [11] which is described as follows.

Proposition 3.1 (Kharitonov [11]). *The polynomials*

$$p(s)=\sum_{i=0}^{n-1} a_i s^i+s^n, \tag{3.1}$$

$a_i \in [x_i, y_i]$, $x_i \le y_i$, $i=0, 1, \ldots, n-1$, where the real coefficients a_i, $i=0, \ldots, n-1$, take any arbitrary value in the closed interval $[x_i, y_i]$, are strictly Hurwitz iff the following four polynomials are strictly Hurwitz:

$$p_1(s)=s^n+x_{n-1}s^{n-1}+x_{n-2}s^{n-2}+y_{n-3}s^{n-3}+y_{n-4}s^{n-4}+x_{n-5}s^{n-5}+x_{n-6}s^{n-6}+\cdots, \tag{3.2}$$

$$p_2(s)=s^n+y_{n-1}s^{n-1}+y_{n-2}s^{n-2}+x_{n-3}s^{n-3}+x_{n-4}s^{n-4}+y_{n-5}s^{n-5}+y_{n-6}s^{n-6}+\cdots, \tag{3.3}$$

$$p_3(s)=s^n+x_{n-1}s^{n-1}+y_{n-2}s^{n-2}+y_{n-3}s^{n-3}+x_{n-4}s^{n-4}+x_{n-5}s^{n-5}+y_{n-6}s^{n-6}+\cdots, \tag{3.4}$$

$$p_4(s)=s^n+y_{n-1}s^{n-1}+x_{n-2}s^{n-2}+x_{n-3}s^{n-3}+y_{n-4}s^{n-4}+y_{n-5}s^{n-5}+x_{n-6}s^{n-6}+\cdots. \tag{3.5}$$

For a proof of Proposition 3.1 see also Bose [5]. The proof of Theorem 1.3 now follows from the following argument.

If $k(s)$ given by (1.3) is a positive compensator, every entry of the associated vector $\boldsymbol{\Psi}$ is a non-negative real number. Consider now the set of all characteristic polynomials of the closed loop system $g(s)[1+k(s)g(s)]^{-1}$ where $k(s)$ is a positive compensator and $g(s) \in \mathscr{F}_2$. Since every entry of $\boldsymbol{\Psi}$ is non-negative, it follows that the co-efficient of s^i of the characteristic polynomials belongs to the interval $[\boldsymbol{\Psi}u_i^{\mathrm{T}}, \boldsymbol{\Psi}v_i^{\mathrm{T}}]$, $i=0, 1, \ldots, n-1$. Theorem 1.3 now follows from Proposition 3.1 since the polynomials (1.15)–(1.18) are exactly the associated polynomials (3.2)–(3.5) under this case. □

Proof of Corollary 1.4. The necessity follows from the fact that the associated eight plants belong to $\mathscr{F}_2$. The sufficiency now follows from the following argument. Assume $k > 0$ simultaneously stabilizes the eight plants and therefore in particular stabilizes (1.19)–(1.22). The associated four characteristic polynomials are exactly the polynomials (3.2)–(3.5). Hence by Proposition 3.1, k simultaneously stabilizes every plant in $\mathscr{F}_2$. On the other hand if $k \leq 0$ simultaneously stabilizes the eight plants, one has to work with the plants (1.23)–(1.26) as above. □

Proof of Corollary 1.5. The proof follows clearly from the argument given in Corollary 1.4. □

Proof of Corollary 1.6. First of all we want to show that there exist some x_i, $y_i \in \mathbb{R}$, $i = 0, \ldots, n-1$, such that the polynomials (3.2)–(3.5) are strictly Hurwitz. This however follows from the following argument.

There exist $x_0, \ldots, x_{n-1}$ such that $p_1(s)$ is strictly Hurwitz. By considering $(x_0, \ldots, x_{n-1})$ as a point in $\mathbb{R}^n$ there exists an open neighbourhood of $(x_0, \ldots, x_{n-1})$ in $\mathbb{R}^n$ such that the associated polynomials are all strictly Hurwitz. Choose y_i sufficiently close to x_i for $i = 0, \ldots, n-1$. For this particular choice of x_i and y_i the polynomials p_2, p_3, p_4 are sufficiently close to p_1 and are therefore strictly Hurwitz.

The proof of this corollary now follows from the fact that if (1.11) and (1.12) are simultaneously pole assignable by a positive compensator, there exists a positive vector Ψ such that $\Psi[v_i^{\mathrm{T}}]$, $\Psi[u_i^{\mathrm{T}}]$, $i = 0, \ldots, n+q-1$, are arbitrary. It follows that there exists a positive vector ψ such that (1.15)–(1.18) are strictly Hurwitz. Hence by Theorem 1.3, every plant in $\mathscr{F}_2$ is simultaneously stabilizable. □

4. Conclusion

To conclude, we refer to some recent work by Bialas [2], William, Greschak and Verghese [18] and Barmish [1] on the stability of 'interval polynomials' and 'interval matrices'. The results stated here, we hope, may be extended to include the stabilizability properties of a family of multi-input, multi-output systems.

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References

[1] B.R. Barmish, Invariance of strict Hurwitz property for polynomials with perturbed co-efficients, *IEEE Trans. Automat. Control* **29** (1984) 935–936.

[2] S. Bialas, A necessary and sufficient condition for the stability of interval matrices, *Internat. J. Control* **37** (1983) 717–722.

[3] S. Bialas and J. Garloff, On sets of stable polynomials and stable matrices, Herrn Professor Dr. Karl Nickel Zum 60. Geburtstag Gewidmet, Band 1, Universität Freiburg i. Br. (1984) pp. 53–71.

[4] J.D. Birdwell, D. Castanon and M. Athans, On reliable control system designs with and without feedback reconfigurations, *Proc. of 17th C.D.C. Conf.*, San Diego, CA (1979) 419–426.

[5] N.K. Bose, A system-theoretic approach to stability of sets of polynomials, Preprint.

[6] B.K. Ghosh, Simultaneous stabilization and pole placement of a multimode linear dynamical system, Ph.D. Dissertation, Harvard University (1983).

[7] B.K. Ghosh, Simultaneous partial pole-placement — A new approach to multimode system design, *IEEE Trans. Automat. Control*, submitted.

[8] B.K. Ghosh and C.I. Byrnes, Simultaneous stabilization and simultaneous pole placement by non-switching dynamic compensation, *IEEE Trans. Automat. Control* **28** (1983) 735–741.

[9] P.P. Khargonekar and A. Tannenbaum, Robust stabilization of systems with uncertain parameters, *Proceedings of 23rd Conference on Decision and Control*, Las Vegas, NV (1984) 1214–1218.

[10] P.P. Khargonekar and A. Tannenbaum, Non-Euclidean metrics and the robust stabilization of systems with parameter uncertainty, *IEEE Trans. Automat. Control*, to appear.

[11] V.L. Kharitonov, Asymptotic stability of an equilibrium position of a family of systems of linear differential equations, *Differencial'nye Uravnenija* **14** (11) (1978) 2086–2088.

[12] H. Kimura, Robust stabilizability for a class of transfer functions, *IEEE Trans. Automat. Control* **29**(9) (1984) 788–793.

[13] R. Saeks and J.J. Murray, Fractional representation, algebraic geometry and the simultaneous stabilization problem, *IEEE Trans. Automat. Control* **27** (1982) 895–903.

[14] A. Tannenbaum, Feedback stabilization of linear dynamical plants with uncertainty in the gain factor, *Internat. J. Control* **32** (1980) 1–16.

[15] A. Tannenbaum, *Invariance and System Theory: Algebraic and Geometric Aspects* (Springer, Berlin, 1981).

[16] M. Vidyasagar and H. Kimura, Robust controllers for uncertain linear multivariable systems, *9th IFAC World Congress*, Budapest (1984).

[17] M. Vidyasagar and N. Viswanadham, Algebraic design techniques for reliable stabilization, *IEEE Trans. Automat. Control* **27** (1982) 1085–1095.

[18] C.K. William, J.P. Greschak and G.C. Verghese, Comments on "A necessary and sufficient condition for the stability of interval matrices", *Internat. J. Control* **39** (4) (1984) 849–851.