Available online at www.sciencedirect.com
SCIENCE DIRECT
Automatica 40 (2004) 481 – 490
automatica
www.elsevier.com/locate/automatica

Brief paper

# Nonlinear observers for perspective time-invariant linear systems ☆

Rixat Abdursul[a], Hiroshi Inaba[a,*], Bijoy K. Ghosh[b]

[a] Department of Information Sciences, Tokyo Denki University, Hatoyama-machi, Hiki-gun, Saitama 350-0394, Japan
[b] Department of Electrical and Systems Engineering, Washington University, USA

Received 18 November 2002; received in revised form 28 July 2003; accepted 30 October 2003

**Abstract**

Perspective dynamical systems arise in machine vision, in which only perspective observation is available. This paper proposes and studies a Luenberger-type nonlinear observer for perspective time-invariant linear systems. Assuming a given perspective time-invariant linear system to be Lyapunov stable and to satisfy some sort of detectability condition, it is shown that the estimation error converges exponentially to zero. Finally, some simple numerical examples are presented to illustrate the result obtained.


*Keywords:* Perspective system; Nonlinear observer; Luenberger-type observer; Exponential stability; Detectability; Machine vision

## 1. Introduction

The essential problem in dynamic machine vision is how to determine the position of a moving rigid body and/or any unknown parameters characterizing the motion and shape of the body from knowledge of the associated optical flow, and perspective dynamical systems arise in mathematically describing such dynamic machine vision problems. Perspective dynamical systems have been studied in a number of approaches in the framework of systems theory (see, e.g., Abdursul, Inaba, & Ghosh, 2001; Ghosh & Loucks, 1995; Ghosh, Jankovic, & Wu, 1994; Ghosh, Inaba, & Takahashi, 2000; Dayawansa, Ghosh, Martin, & Wang, 1993; Inaba, Yoshida, Abdursul, & Ghosh, 2000; Matveev, Hu, Frezza, & Rehbinder, 2000; Soatto, Frezza, & Perona, 1996; Jankovic & Ghosh, 1995; Ghosh & Rosenthal, 1995). One of interesting approaches discussed in the recent papers is to formulate such a problem by introducing a notion of implicit systems in order to take into account the constraints intrinsically involved in dynamic machine vision (Matveev et al., 2000; Soatto et al., 1996). In particular, Matveev et al. (2000). studied an observer to estimate the unknown state of such an implicit system. Their work turns out to be closely related to the present work, and this relationship will be explained later.

First, following our previous work Abdursul et al., 2001, we introduce a *perspective time-invariant linear system* considered in this study as follows:

$$\dot{x}(t) = Ax(t) + v(t), \qquad x(0) = x_0 \in \boldsymbol{R}^n,$$

$$y(t) = h(Cx(t)), \tag{1}$$

where $x(t) \in \boldsymbol{R}^n$ is the state, $v(t) \in \boldsymbol{R}^n$ the external input, $y(t) \in \boldsymbol{R}^m$ the *perspective observation* of the state $x(t)$, $A \in \boldsymbol{R}^{n\times n}$, $C \in \boldsymbol{R}^{(m+1)n}$ are matrices with $m < n$ and rank $C = m + 1$, and finally $h : \boldsymbol{R}^{m+1} \to \boldsymbol{R}^m$ is a nonlinear function that produces the perspective observation and has the form

$$h(\xi) := \left[\frac{\xi_1}{\xi_{m+1}} \cdots \frac{\xi_m}{\xi_{m+1}}\right]^{\mathrm{T}}, \qquad \xi_{m+1} \neq 0,$$

$$\xi = [\xi_1 \cdots \xi_m \quad \xi_{m+1}]^{\mathrm{T}} \in \boldsymbol{R}^{m+1} \tag{2}$$

It is easily seen that a typical three-dimensional vision problem considered in the previous works (e.g., Abdursul et al., 2001; Ghosh & Loucks, 1995; Ghosh et al., 1994; Ghosh et al., 2000; Dayawansa et al., 1993; Inaba et al., 2000; Matveev et al., 2000; Soatto et al., 1996; Jankovic & Ghosh, 1995) can be described as a special case of the

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Torkel Glad under the direction of Editor Hassan Khalil.

* Corresponding author. Tel.: +81-49-296-2911; fax: +81-49-496-6403.

*E-mail addresses:* rixat@j.dendai.ac.jp (R. Abdursul), inaba@cck.dendai.ac.jp (H. Inaba), ghosh@netra.wustl.edu (B.K. Ghosh).

0005-1098/$ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2003.10.017

perspective linear system (1). In fact, since in such a vision problem the perspective observation is given essentially in the form

$$y=[y_1 \quad y_2]^{\mathrm{T}}=[x_1/x_3 \quad x_2/x_3]^{\mathrm{T}},$$

where $x=[x_1 \quad x_2 \quad x_3]^{\mathrm{T}}$ represents the coordinate of a feature point in question, one can set $n=3$, $m=2$ and $C=I_3$ (the identity matrix) to describe the perspective observation in the form $y=h(Cx)$ as in (1).

Now, we briefly explain the relationship between our perspective system of form (1) and an implicit system introduced in Matveev et al. (2000). The implicit system is described in the following form:

$$\dot{\xi}(t)=A\xi(t), \quad \xi(0)=\xi_0\in\boldsymbol{R}^n,$$

$$B(y(t))\xi(t)=p(t), \tag{3}$$

where $\xi(t)\in\boldsymbol{R}^n$ is the state, $B:\boldsymbol{R}^m \to \boldsymbol{R}^{l\times n}$ a continuous mapping, $p(t)\in\boldsymbol{R}^l$ a known function and finally $y(t)\in\boldsymbol{R}^m$ an implicitly measured output that is determined from $\xi(t)$ so as to satisfy the equation $B(y(t))\xi(t)=p(t)$. Then it is shown in Matveev et al. (2000) that under suitable assumptions on (3) it is possible to design a Luenberger-type observer of the following form:

$$\frac{\mathrm{d}\hat{\xi}(t)}{\mathrm{d}t}=A\hat{\xi}(t)-P^{-1}B(y(t))*[B(y(t))\hat{\xi}(t)-p(t)] \tag{4}$$

such that the estimation error $\xi(t)-\hat{\xi}(t)$ converges exponentially to zero, where $P$ is a suitably chosen matrix. Further it is shown that a system of form (1) can be transformed into an implicit system of form (3) by the coordinate transformation

$$\xi(t)=x(t)-\int_0^t \mathrm{e}^{A(t-s)}v(s)\,\mathrm{d}s \tag{5}$$

and by setting $l:=m$ and

$$B(y):=[-I \quad y]C,$$

$$p(t):=-B(y(t))\int_0^t \mathrm{e}^{A(t-s)}v(s)\,\mathrm{d}s. \tag{6}$$

The purpose of the present paper is to investigate a Luenberger-type observer for system (1) without converting it to an implicit system of form (3). The advantage of this observer over the one in Matveev et al. (2000) is obvious since the former does not require the trasformation (5) that involves the integration of the input $v(s)$ $(0\leqslant s\leqslant t)$. This integration may cause difficulty in implementation, in particular, for the case that the input $v(t)$ is given in a state feedback form in which the integration must be performed in real time.

The present paper is organized as follows: First in Section 2, a *nonlinear observer of the Luenberger-type* for system (1) is proposed with some preliminary results and the main theorem is stated. Then in Section 3 the main theorem is proved. More precisely, it is shown that, under suitable assumptions on system (1), including that it is Lyapunov stable and satisfies some sort of detectability condition, the estimation error converges exponentially to zero. In Section 4, two simple numerical examples are presented to illustrate the proposed nonlinear observer and its convergence property. The numerical results show that the observer works well. Finally, Section 5 gives some concluding remarks.

## 2. Luenberger-type nonlinear observers

In this section, we propose a Luenberger-type observer for a perspective linear system of form (1), and present our main theorem.

First, notice that, denoting an estimate of the state $x(t)$ by $\hat{x}(t)$, a full-order state observer for system (1) generally has the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{x}(t)=\varphi(\hat{x}(t),v(t),y(t)), \quad \hat{x}(0)=\hat{x}_0\in\boldsymbol{R}^n \tag{7}$$

and satisfies the condition that whenever $\hat{x}(0)=x(0)$ the solution $\hat{x}(t)$ of (7) coincides completely with the solution $x(t)$ of system (1) for any $v(\cdot)$. Therefore, under the condition that (7) has a unique solution, it is possible to assume that the function $\varphi(\hat{x},v,y)$ has the form

$$\varphi(\hat{x},v,y)=A\hat{x}+v+r(\hat{x},y),$$

where $r(\hat{x},y)$ is any sufficiently smooth function satisfying the condition $r(x,h(Cx))=0$ for all $x\in\boldsymbol{R}^n$. Among many functions $r(\hat{x},y)$ satisfying this condition, it would be wise to choose a function, which is reasonably simple, but has sufficient freedom to adjust its characteristics. As such a function, it is possible to choose

$$r(\hat{x},y)=K(y,\hat{x})[y-h(C\hat{x})],$$

where $K(y,\hat{x})$ is any sufficiently smooth function. Hence, it is possible to consider an observer of the Luenberger-type given as

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{x}(t)=A\hat{x}(t)+v(t)+K(y(t),\hat{x}(t))[y(t)-h(C\hat{x}(t))],$$

$$\hat{x}(0)=\hat{x}_0\in\boldsymbol{R}^n, \tag{8}$$

where $K(y,\hat{x})$ is a suitable matrix-valued function $K:\boldsymbol{R}^m\times\boldsymbol{R}^n\to\boldsymbol{R}^{n\times m}$, called an *observer gain matrix*.

The next important step is how to choose a gain matrix $K(y,\hat{x})$ in (8). To do so, first letting

$$\xi=[\xi_1\cdots\xi_m \quad \xi_{m+1}]^{\mathrm{T}}:=Cx,$$

$$\hat{\xi}=[\hat{\xi}_1\cdots\hat{\xi}_m \quad \hat{\xi}_{m+1}]^{\mathrm{T}}:=C\hat{x},$$

$$C=[C_1^{\mathrm{T}}\cdots C_m^{\mathrm{T}} \quad C_{m+1}^{\mathrm{T}}]^{\mathrm{T}}\in\boldsymbol{R}^{(m+1)\times n}$$

and using system (1) and (2), one can easily obtain

$$y-h(C\hat{x})$$

$$=h(Cx)-h(C\hat{x})$$

$$=\left[\frac{\xi_1}{\xi_{m+1}}-\frac{\hat{\xi}_1}{\hat{\xi}_{m+1}}\cdots\frac{\xi_m}{\xi_{m+1}}-\frac{\hat{\xi}_m}{\hat{\xi}_{m+1}}\right]^{\mathrm{T}}$$

$$
= \frac{1}{C_{m+1}\hat{x}} \begin{bmatrix} 1 & 0 & \cdots & 0 & -y_1 \\ 0 & 1 & \cdots & 0 & -y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -y_m \end{bmatrix} \begin{bmatrix} C_1(x-\hat{x}) \\ C_2(x-\hat{x}) \\ \vdots \\ C_{m+1}(x-\hat{x}) \end{bmatrix}
$$

$$
= \frac{1}{C_{m+1}\hat{x}} B(y)C\rho, \tag{9}
$$

where $B(y)$ is the matrix-valued function given by

$$
B(y) := [I_m \quad -y] \in \boldsymbol{R}^{m(m+1)} \tag{10}
$$

and $\rho \in \boldsymbol{R}^n$ is the estimation error vector defined by

$$
\rho := x - \hat{x}. \tag{11}
$$

Now, using (1) and (8) together with (9),

$$
\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \left\{ A - \frac{1}{C_{m+1}\hat{x}(t)} K(y(t),\hat{x}(t))B(y(t))C \right\} \times \rho(t), \tag{12}
$$

where $\rho(t_0) = x(0) - \hat{x}(0) \in \boldsymbol{R}^n$. To eliminate the singularity in (12), let us choose a gain matrix $K(y,\hat{x})$ of the form

$$
K(y,\hat{x}) := C_{m+1}\hat{x}P^{-1}C^*B^*(y), \tag{13}
$$

where $P \in \boldsymbol{R}^{n\times n}$ is an appropriately chosen matrix, which is considered to be a free parameter for the gain matrix.

Next, we make various assumptions on system (1), which seem to be necessary and/or reasonable from the viewpoint of machine vision.

**Assumption 2.1.** System (1) is assumed to satisfy the following conditions:

(i) System (1) is Lyapunov stable.
(ii) The observation vector $y(t)$ is a continuous and bounded function of $t$, that is,

$$
y(\cdot) \in C^m[0,\infty) \cap L^m_\infty[0,\infty). \tag{14}
$$

(iii) Express the set $\sigma(A)$ of all eigenvalues of $A$ as $\sigma(A) = \sigma_-(A) \cup \sigma_0(A)$ where $\sigma_-(A)$ and $\sigma_0(A)$ indicate the sets of eigenvalues with strictly negative real part and zero real part, respectively. Let, $W_-, W_0 \subset \boldsymbol{C}^n$ denote the generalized eigenspaces corresponding to $\sigma_-(A)$ and $\sigma_0(A)$, respectively, and choose a basis matrix $E_0 = [\xi_1 \ \cdots \ \xi_r]$ for $W_0$ where $r := \dim W_0$. Then, there exist $T > 0$ and $\varepsilon > 0$ such that

$$
\int_0^T E_0^* \mathrm{e}^{A^*\tau} C^* B^*(y(t+\tau)) \times B(y(t+\tau))C\mathrm{e}^{A\tau}E_0 \mathrm{d}\tau \geqslant \varepsilon I_r, \quad \forall t \geqslant 0. \tag{15}
$$

**Remark 2.2.** All the conditions given in Assumption 2.1 are necessary and/or reasonable requirements from the viewpoint of machine vision.

(i) Condition (i) is imposed to ensure that if $v(t) \equiv 0$ then the motion of a moving body takes place within a bounded space.
(ii) Condition (ii) is imposed to ensure that the motion is smooth enough and takes place inside a conical region centered at the camera to produce a continuous and bounded measurement $y(t)$ on the image plane.
(iii) Condition (iii) is imposed to ensure some sort of detectability for the perspective system (1). In fact, inequality (15) implies that $(C,A)$ is a detectable pair and the external input $v(t)$ must not be identically zero. These facts are verified in Proposition 2.3.

The following proposition gives some system theoretical implications of Assumption 2.1(iii).

**Proposition 2.3.** *Assume that system* (1) *is Lyapunov stable, let $A_0$ denote the critically stable part* (*i.e., the part corresponding to $\sigma_0(A)$*) *of $A$ and set $C_0 := CE_0$. If Assumption* 2.1(iii) *is satisfied, then the following statements hold true*:

(i) *$(C,A)$ is a detectable pair, that is, the critically stable part $(C_0,A_0)$ of $(C,A)$ is observable.*
(ii) *The external input $v(t)$ is never identically zero.*

**Proof.** To prove (i), first recall that the pair $(C,A)$ being detectable is equivalent to its critically stable part $(C_0,A_0)$ being observable. Therefore, it suffices to show that (15) implies,

$$
M := \int_0^T E_0^* \mathrm{e}^{A^*\tau} C^* C \mathrm{e}^{A\tau} E_0 \,\mathrm{d}\tau
$$

$$
= \int_0^T \mathrm{e}^{A_0^*\tau} C_0^* C_0 \mathrm{e}^{A_0\tau} \,\mathrm{d}\tau \geqslant \varepsilon_0 I_r \tag{16}
$$

for some $\varepsilon_0 > 0$. To show this, first note that

$$
C\mathrm{e}^{At}E_0 = CE_0\mathrm{e}^{A_0 t} = C_0 \mathrm{e}^{A_0 t} \tag{17}
$$

from which the second equality in (16) follows. In order to show the last inequality in (16), it suffices to verify that $M$ is strictly positive-definite. To do so, we show that if $M$ is not strictly positive-definite then (15) is not satisfied. So assume that $M$ is not strictly positive-definite. Then, there exists a nonzero vector $\xi \in \boldsymbol{C}^r$ such that

$$
\int_0^T \xi^* \mathrm{e}^{A_0^*\tau} C_0^* C_0 \mathrm{e}^{A_0\tau}\xi \,\mathrm{d}\tau = \int_0^T \|C_0 \mathrm{e}^{A_0\tau}\xi\|^2 \,\mathrm{d}\tau = 0,
$$

which leads to

$$
C_0 \mathrm{e}^{A_0\tau}\xi \equiv 0 \in C^{m+1}. \tag{18}
$$

Therefore, it follows from (17) and (18) that

$$\int_0^T \xi^* E_0^* e^{A^*\tau} C^* B^*(y(t+\tau)) B(y(t+\tau)) C e^{A\tau} E_0 \xi \, d\tau$$

$$= \int_0^T \|B(y(t+\tau)) C_0 e^{A_0\tau} \xi\|^2 \, d\tau = 0,$$

showing that (15) is not satisfied.

Next, to prove (ii), first consider the critically stable subsystem of (1) given by

$$\frac{d}{d\tau}\xi(t) = A_0 \xi(t), \quad \xi(0) = \xi_0 \in \boldsymbol{C}^r,$$

$$\eta(t) = h(C_0 \xi(t)), \tag{19}$$

where $\xi_0$ represents the critically stable initial vector corresponding to the initial state $x_0 \in \boldsymbol{R}^n$ of system (1). Noticing that the critically stable state is given by $\xi(t) = e^{A_0 t}\xi_0$, partition the vector $C_0\xi(t)$ as follows:

$$C_0\xi(t) = C_0 e^{A_0 t}\xi_0 = \begin{bmatrix} \hat{\mu}(t) \\ \mu_{m+1}(t) \end{bmatrix} \in \boldsymbol{C}^{m+1}, \quad \hat{\mu}(t) \in \boldsymbol{C}^m.$$

Then, since $\eta(t) = h(C_0\xi(t)) = \hat{\mu}(t)/\mu_{m+1}(t)$, using (17) and $B(y) = [I_m \quad -y]$ given by (10), one obtains

$$Q(\xi(t)) := \int_0^T \xi(t) E_0^* e^{A^*\tau} C^* B^*(y(t+\tau)) \times B(y(t+\tau)) C e^{A\tau} E_0 \xi(t) \, d\tau$$

$$= \int_0^T \|B(y(t+\tau)) C_0 \xi(t+\tau)\|^2 \, d\tau$$

$$= \int_0^T \|\hat{\mu}(t+\tau) - \mu_{m+1}(t+\tau) y(t+\tau)\|^2 \, d\tau$$

$$= \int_0^T |\mu_{m+1}(t+\tau)|^2 \|\eta(t+\tau) - y(t+\tau)\|^2 \, d\tau.$$

Now, consider the measurement $y(t)$ in system (1) with $v(t) \equiv 0$ and $x_0 \neq 0$ such that $\xi_0 \neq 0$. First, note that $\|\xi(t)\| = \|\xi_0\| \neq 0$ for all $t \geqslant 0$ and $|\mu_{m+1}(t+\tau)|$ is bounded because all the eigenvalues of $A_0$ are purely imaginary. Further note that $y(t)$ converges to the measurement $\eta(t)$ in the critically stable subsystem (19) because the contribution from the stable subsystem of (1) to $y(t)$ approaches zero. Therefore, $Q(\xi(t)) \to 0$ $(t \to \infty)$, and hence for a sufficiently large $t > 0$, one obtains

$$Q(\xi(t)) = \int_0^T |\mu_{m+1}(t+\tau)|^2 \|\eta(t+\tau) - y(t+\tau)\|^2 \, d\tau \leqslant \varepsilon \|\xi(t)\|^2 = \varepsilon \|\xi_0\|^2,$$

which implies that if $v(t) \equiv 0$ (15) is never satisfied. □

Inequality (15) requires that not only the pair $(C, A)$ is detectable, but also that $v(t)$ must not be identically zero. It is worthwhile to mention that the present authors (Inaba et al., 2000) have already shown that for a very simple perspective linear system the condition that $v(t)$ is not identically zero is necessary for observability. Further, it should be recalled that, for a general linear system with system matrix $A$ and output matrix $C$, the detectability is completely determined by the pair $(C, A)$, and completely independent of the input $v(t)$ and initial state $x(0)$. On the other hand, for a general nonlinear system the detectability is generally dependent on the input and initial state or equivalently the trajectory generated by the input and initial state. Therefore in order to check the detectability of a nonlinear system it is inevitable to verify a similar condition like (15) along the trajectory, but such verification may not be an easy task, as seen in verifying if inequality (15) is satisfied for a given trajectory $x(t)$. In fact there seems to be impossible to develop a reasonably ease and practical method for checking inequality (15).

Before stating our main theorem, the following lemma is proved.

**Lemma 2.4.** *Let $A \in \boldsymbol{C}^{n\times n}$ be a Lyapunov stable matrix, and consider the linear system of differential equations*

$$\dot{x}(t) = Ax(t), \quad x(0) = x_0 \in \boldsymbol{C}^n. \tag{20}$$

*Further, use the same notations as in Assumption* 2.1(iii) *and let $\pi_- : \boldsymbol{C}^n \to W_-$, $\pi_0 : \boldsymbol{C}^n \to W_0$ denote the matrix representations of the projection operators along $W_0$, $W_-$, respectively.*

*Then, for any $a > 0$ the matrix inequality*

$$PA + A^*P \leqslant -a\pi_-^*\pi_- \tag{21}$$

*has a positive-definite Hermitian solution $P \in \boldsymbol{C}^{n\times n}$.*

**Proof.** First, note that the Lyapunov stability of $A$ implies that each $\lambda \in \sigma_0(A)$ has a unity multiplicity. Now, consider the generalized eigenspaces $W_-, W_0$, and their basis matrices $E_- \in \boldsymbol{C}^{n\times q}$, $E_0 \in \boldsymbol{C}^{n(n-q)}$ where $q := \dim W_-$, and set $E := [E_- \;\; E_0] \in \boldsymbol{C}^{n\times n}$. Then, the matrix $A$ is decomposed by a similarity transformation $E$ into a direct sum of a stable sub-matrix $A_-$ and a critically stable sub-matrix $A_0$ in the new basis $E$, that is, one obtains

$$\hat{A} := E^{-1}AE = \begin{bmatrix} A_- & 0 \\ 0 & A_0 \end{bmatrix}, \quad A_- \in \boldsymbol{C}^{q\times q},$$

$$A_0 \in \boldsymbol{C}^{(n-q)\times(n-q)}. \tag{22}$$

Further, since all the column vectors of $E_0$ belong to $W_0$, the projection matrix $\pi_-$ is transformed to

$$\hat{\pi}_- := \pi_- E = \pi_-[E_- \;\; E_0] = [\pi_- E_- \;\; 0] \in \boldsymbol{C}^{n\times n}. \tag{23}$$

Now, it is easy to see that inequality (21) is equivalent to $\hat{P}\hat{A} + \hat{A}^*\hat{P} \leqslant -a\hat{\pi}_-^*\hat{\pi}_-$ with $\hat{P} := E^*PE$. In fact, (21) is equivalent to the following matrix inequality:

$$\begin{bmatrix} P_- & P_z \\ P_z^* & P_0 \end{bmatrix}\begin{bmatrix} A_- & 0 \\ 0 & A_0 \end{bmatrix} + \begin{bmatrix} A_-^* & 0 \\ 0 & A_0^* \end{bmatrix}\begin{bmatrix} P_- & P_z \\ P_z^* & P_0 \end{bmatrix} \leqslant -a\begin{bmatrix} E_-^*\pi_-^*\pi_- E_- & 0 \\ 0 & 0 \end{bmatrix} \tag{24}$$

which is easily reduced to

$$\begin{bmatrix} P_-A_- + A_-^*P_- & P_zA_0 + A_-^*P_z \\ P_z^*A_- + A_0^*P_z^* & P_0A_0 + A_-^*P_0 \end{bmatrix} \leqslant -a \begin{bmatrix} E_-^*\pi_-^*\pi_-E_- & 0 \\ 0 & 0 \end{bmatrix}. \tag{25}$$

Since $Q_- := aE_-^*\pi_-^*\pi_-E_-$ is positive-definite and Hermitian, the $(1,1)$th inequality in (25), i.e.,

$$P_-A_- + A_-^*P_- \leqslant -Q_-$$

has a solution $P_- = P_-^* > 0$ because, since $A_-$ is stable, at least equation $P_-A_- + A_-^*P_- = -Q_-$ gives such a solution. Further, the $(2,2)$th inequality $P_0A_0 + A_0^*P_0 \leqslant 0$ has also a solution $P_0 = P_0^* > 0$ because $A_0$ is Lyapunov stable. Finally, the $(1,2)$th and $(2,1)$th inequalities are satisfied with $P_z = 0$. Therefore (24) has a positive-definite Hermitian solution $\hat{P} = \mathrm{diag}\{P_-, P_0\}$, and hence (21) has a positive-definite Hermitian solution $P = (E^{-1})^*\hat{P}E^{-1}$. This completes the proof. □

Now, it is ready to state our main theorem. However, since its proof requires a number of lengthy and cumbersome technical arguments, the proof is given in the next section.

**Theorem 2.5** (Nonlinear observers). *Assume that system (1) satisfies Assumption 2.1 and consider a nonlinear observer of the Luenberger-type (8), that is,*

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{x}(t) = A\hat{x}(t) + v(t) + K(y(t),\hat{x}(t))[y(t) - h(C\hat{x}(t))],$$

$$\hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n, \tag{26}$$

*where the gain matrix is given by (13), that is,*

$$K(y,\hat{x}) := C_{m+1}\hat{x}P^{-1}\boldsymbol{C}^*B^*(y) \tag{27}$$

*and $B(y)$ is given by (10). Further, let $\pi_- : \boldsymbol{C}^n \to W_-$, $\pi_0 : \boldsymbol{C}^n \to W_0$ denote the matrix representations of the projection operators along $W_0$, $W_-$, respectively, and $P \in \boldsymbol{R}^{n\times n}$ be a symmetric positive definite matrix satisfying the Lyapunov inequality*

$$A^*P + PA \leqslant -a\pi_-^*\pi_-, \tag{28}$$

*where $a > 0$ is a constant.*

*Then, the following statements hold:*

(i) *The estimation error $\rho(t) := x(t) - \hat{x}(t)$ satisfies the differential equation*

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = [A - P^{-1}C^*B^*(y(t))B(y(t))C]\rho(t),$$

$$\rho(0) \in \boldsymbol{R}^n. \tag{29}$$

(ii) *$\rho(t)$ converges exponentially to zero, that is, there exist $\alpha > 0$, $\beta > 0$ such that*

$$\|\rho(t)\| := \|x(t) - \hat{x}(t)\| \leqslant \beta \mathrm{e}^{-\alpha t}\|\rho(0)\|, \quad \forall t \geqslant 0. \tag{30}$$

## 3. Proof of Theorem 2.5

Because its proof requires quite lengthy and cumbersome technical arguments, first two important lemmas will be proved in the following subsection.

### 3.1. Preliminaries

First of all, we deduce several important facts from the given assumptions. Using the notations in Theorem 2.5, define

$$\rho_-(t) := \pi_-\rho(t), \quad \rho_0(t) := \pi_0\rho(t), \quad \forall t \geqslant 0. \tag{31}$$

Then since $C^n = W_- \oplus W_0$ one has

$$\rho(t) = \rho_-(t) + \rho_0(t) \tag{32}$$

which easily leads to the inequality

$$\int_0^\infty \|\rho(t)\|^2\,\mathrm{d}t = \int_0^\infty \|\rho_-(t) + \rho_0(t)\|^2\,\mathrm{d}t \leqslant 2\int_0^\infty \|\rho_-(t)\|^2\,\mathrm{d}t + 2\int_0^\infty \|\rho_0(t)\|^2\,\mathrm{d}t. \tag{33}$$

Further the following lemma is easily proved.

**Lemma 3.1.** *The following inequalities hold true*:

$$\rho^*(t)P\rho(t) \leqslant \rho^*(0)P\rho(0), \quad t \geqslant 0$$

$$a\int_0^t \|\rho_-(s)\|^2\,\mathrm{d}s \leqslant \rho^*(0)P\rho(0), \quad \forall t \geqslant 0$$

$$2\int_0^t \|B(y(s))C\rho(s)\|^2\,\mathrm{d}s \leqslant \rho^*(0)P\rho(0), \quad \forall t \geqslant 0. \tag{34}$$

**Proof.** Using (29), (28) and (31), one can easily obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho(t)^*P\rho(t)) = \rho(t)^*(A^*P + PA)\rho(t) - 2\rho^*(t)C^*B^*(y(t))B(y(t))C\rho(t) \leqslant -a\|\rho_-(t)\|^2 - 2\|B(y(t))C\rho(t)\|^2$$

and hence, for any $t \geqslant 0$,

$$0 \leqslant \rho^*(t)P\rho(t) \leqslant \rho^*(0)P\rho(0) - a\int_0^t \|\rho_-(s)\|^2\,\mathrm{d}s - 2\int_0^t \|B(y(s))C\rho(s)\|^2\,\mathrm{d}s$$

which leads to the desired inequalities. □

Before proving the next lemma, we note that it easily follows using (10) and Assumption 2.1(ii) that

$$\|B(y(\cdot))\|_\infty = \sup_{t\geqslant 0}\|[I_m \quad -y(t)]\| \leqslant 1 + \|y(\cdot)\|_\infty \tag{35}$$

which is used in the squel.

**Lemma 3.2.** *Consider the solution* $\rho(t)$ *of* (29) *and* $\rho_-(t)$, $\rho_0(t)$ *defined in* (31).

(i) *There exists a constant* $\gamma_- > 0$, *independent of* $\rho(0)$, *such that*

$$\int_0^\infty \|\rho_-(t)\|^2 \, dt \leqslant \gamma_- \|\rho(0)\|^2.$$

(ii) *There exists a constant* $\gamma_0 > 0$, *independent of* $\rho(0)$, *such that*

$$\int_0^\infty \|\rho_0(t)\|^2 \, dt \leqslant \gamma_0 \|\rho(t_0)\|^2.$$

**Proof.** Statement (i) is a direct consequence of the second inequality in Lemma 3.1. In fact, since $P^* = P > 0$ and hence $\rho^*(0)P\rho(0) \leqslant \|P\| \, \|\rho(0)\|^2$, one easily obtains the desired inequality

$$\int_0^\infty \|\rho_-(t)\|^2 \, dt \leqslant \frac{\|P\|}{a} \|\rho(0)\|^2 =: \gamma_- \|\rho(0)\|^2,$$

where $\gamma_- > 0$ is independent of $\rho(0)$.

Next statement (ii) is proved. First, notice that the generalized eigenspace $W_0$ is $A$-invariant, so that $\pi_0 A \rho(t) = A\pi_0\rho(t) = A\rho_0(t)$. Using this fact and (29), one obtains

$$\rho_0(t) = \pi_0 \dot{\rho}(t) = A\rho_0(t) - \theta(t), \tag{36}$$

where

$$\theta(t) := \pi_0 P^{-1} C^* B^*(y(t)) B(y(t)) C \rho(t) \in L_2^n[0,\infty) \tag{37}$$

and the fact $\theta(t) \in L_2^n[0,\infty)$ follows from (35) and the third inequality in Lemma 3.1. So the solution of (36) is represented as

$$\rho_0(t+\tau) = e^{A\tau}\rho_0(t) - \int_t^{t+\tau} e^{A(t+\tau-\sigma)}\theta(\sigma) \, d\sigma,$$
$$\forall t, \tau \geqslant 0. \tag{38}$$

Now, consider the basis matrix $E_0 = [\xi_1 \ \cdots \ \xi_r]$ for $W_0$ given in Assumption 2.1(iii) and its generalized inverse matrix $E_0^\dagger := (E_0^* E_0)^{-1} E_0^* \in \boldsymbol{C}^{r\times n}$. Then, since any $x \in W_0$ is uniquely represented as $x = E_0 \xi$ for some $\xi \in \boldsymbol{C}^r$, it follows that

$$E_0 E_0^\dagger x = E_0 (E_0^* E_0)^{-1} E_0^* E_0 \xi = x, \quad \forall x \in W_0.$$

Therefore, recalling $\rho_0(t) \in W_0$, the above equality implies

$$E_0 E_0^\dagger \rho_0(t) = \rho_0(t), \quad \forall t \geqslant 0,$$

which together with Assumption 2.1(iii) and (38) leads to

$$\int_{t_0}^\infty \|\rho_0(t)\|^2 \, dt = \int_0^\infty \|E_0 E_0^\dagger \rho_0(t)\|^2 \, dt$$
$$\leqslant \|E_0\|^2 \int_0^\infty \|E_0^\dagger \rho_0(t)\|^2 \, dt \quad (\|E_0\| = 1)$$
$$= \frac{1}{\varepsilon} \int_0^\infty \rho_0^*(t) E_0^{\dagger *} (\varepsilon I_r) E_0^\dagger \rho_0(t) \, dt$$
$$\leqslant \frac{1}{\varepsilon} \int_0^\infty \left[ \int_0^T \left\| B(y(t+\tau)) C \left\{ \rho_0(t+\tau) + \int_t^{t+\tau} e^{A(t+\tau-\sigma)}\theta(\sigma) \, d\sigma \right\} \right\|^2 d\tau \right] dt$$
$$\leqslant \frac{2}{\varepsilon} (Q_1 + Q_2), \tag{39}$$

where

$$Q_1 := \int_0^\infty dt \int_0^T \|B(y(t+\tau)) C \rho_0(t+\tau)\|^2 \, d\tau,$$

$$Q_2 := \int_0^\infty dt \int_0^T \left\| B(y(t+\tau)) C \int_t^{t+\tau} e^{A(t+\tau-\sigma)}\theta(\sigma) d\sigma \right\|^2 d\tau.$$

Next, $Q_1$ and $Q_2$ are evaluated. Before doing so, an inequality is shown for later usages. Consider a function $f(t) \in L_2(\boldsymbol{R})$, and let $t_0 \in \boldsymbol{R}$ and $T > 0$. Then

$$\int_0^\infty dt \int_t^{t+T} f(\tau) \, d\tau$$
$$= \int_{t_0}^{t_0+T} \int_t^\tau f(\tau) \, dt \, d\tau + \int_{t_0+T}^\infty \int_{\tau-T}^\tau f(\tau) \, dt \, d\tau$$
$$= \int_{t_0}^{t+T} f(\tau)(\tau - t) \, d\tau + \int_{t_0+T}^\infty f(\tau)(\tau - \tau + T) \, d\tau$$
$$\geqslant T \int_{t_0}^{t+T} f(\tau) \, d\tau + T \int_{t_0+T}^\infty f(\tau) \, d\tau$$
$$= T \int_{t_0}^\infty f(\tau) \, d\tau. \tag{40}$$

Now, to evaluate $Q_1$, noticing from (34) that

$$2 \int_0^\infty \|B(y(s)) C \rho(s)\|^2 \, ds \leqslant \rho^*(0) P \rho(0)$$
$$\leqslant \|P\| \|\rho(0)\|^2 \tag{41}$$

and using (40), (32), (41), (35) and Lemma 3.2(i), one obtains

$$Q_1 = \int_0^\infty dt \int_0^T \|B(y(t+\tau)) C \rho_0(t+\tau)\|^2 \, d\tau$$
$$\leqslant T \int_0^\infty \|B(y(t)) C \rho_0(t)\|^2 \, dt$$
$$\leqslant 2T \int_0^\infty \|B(y(t)) C \rho(t)\|^2 \, dt$$

$$+2T\int_0^\infty \|B(y(t))C\rho_-(t)\|^2\,\mathrm{d}t$$

$$\leqslant T\{\|P\|+2(1+\|y(\cdot)\|_\infty)^2\|C\|^2\gamma_-\}\|\rho(0)\|^2$$

$$=:\gamma_{01}\|\rho(0)\|^2, \tag{42}$$

where $\gamma_{01}>0$ is a constant, independent of $\rho(0)$.

Next, to evaluate $Q_2$, first note that, since by Assumption 2.1(i) $A$ is Lyapunov stable, there is a constant $b>0$ such that $\|e^{At}\|\leqslant b<\infty$ for all $t\geqslant 0$. Using this fact, one can proceed to evaluating

$$Q_2=\int_0^\infty\left\{\int_0^T \|B(y(t+\tau))C \times \int_t^{t+\tau} e^{A(t+\tau-\sigma)}\theta(\sigma)\,\mathrm{d}\sigma\right\|^2\mathrm{d}\tau\right\}\mathrm{d}t$$

$$\leqslant (1+\|y(\cdot)\|_\infty)^2\|C\|^2 \times \int_0^\infty\left\{\int_0^T\left\|\int_t^{t+\tau} e^{A(t+\tau-\sigma)}\theta(\sigma)\,\mathrm{d}\sigma\right\|^2\mathrm{d}\tau\right\}\mathrm{d}t$$

$$\leqslant (1+\|y(\cdot)\|_\infty)^2\|C\|^2 b^2 \times \int_0^\infty\left[\int_0^T\left\{\int_t^{t+\tau}\|\theta(\sigma)\|\,\mathrm{d}\sigma\right\}^2\mathrm{d}\tau\right]\mathrm{d}t$$

$$\leqslant K\int_0^\infty \mathrm{d}t\int_t^{t+T}\|\theta(\sigma)\|^2\,\mathrm{d}\sigma,$$

where

$$K:=(1+\|y(\cdot)\|_\infty)^2\|C\|^2 b^2T^2.$$

Moreover, using (40) and (37), and further using the fact $\|\pi_{us}\|=1$ and (41), the above expression for $Q_2$ can be further reduced to

$$Q_2\leqslant KT\int_0^\infty\|\theta(t)\|^2\,\mathrm{d}t$$

$$\leqslant \tfrac{1}{2}KT(1+\|y(\cdot)\|_\infty)^2\|C^*\|^2\|P^{-1}\|^2\|P\|\|\rho(0)\|^2$$

$$=:\gamma_{02}\|\rho(0)\|^2, \tag{43}$$

where $\gamma_{02}>0$ is a constant, independent of $\rho(0)$.

Finally, substituting (42) and (43) into (39), one obtains the desired inequality

$$\int_0^\infty\|\rho_0(t)\|^2\,\mathrm{d}t\leqslant\frac{2}{\varepsilon}(\gamma_{01}+\gamma_{02})\|\rho(0)\|^2=:\gamma_0\|\rho(0)\|^2.$$

This completes the proof of Lemma 3.2. □

### 3.2. *Proof of Theorem 2.5(ii)*

The statement (i) in Theorem 2.5 directly follows from (12) and (13). Therefore, only the statement (ii) will be proved. To prove Theorem 2.5(ii), it suffices to establish by virtue of Lemma 5.1.2 in (Curtain & Zwart, 1995) that there exists some $\gamma>0$, independent of $\rho(0)$, such that the solution $\rho(t)$ of (29) satisfies

$$\int_0^\infty\|\rho(t)\|^2\,\mathrm{d}t\leqslant\gamma\|\rho(0)\|^2.$$

But this task is straightforward by using Lemmas 3.2. In fact, it follows from (33) and Lemmas 3.2 that

$$\int_0^\infty\|\rho(t)\|^2\,\mathrm{d}t\leqslant 2\int_0^\infty\|\rho_-(t)\|^2\,\mathrm{d}t+2\int_0^\infty\|\rho_0(t)\|^2\,\mathrm{d}t$$

$$\leqslant 2(\gamma_-+\gamma_0)\|\rho(0)\|^2=:\gamma\|\rho(0)\|^2,$$

where $\gamma>0$ is a constant, independent of $\rho(0)$. This completes the proof of Theorem 2.4 (ii). □

## 4. Computer simulations

In this section, two simple numerical examples are presented to illustrate the nonlinear observer obtained in the previous sections.

**Example 1.** We consider a very simple perspective linear system of the form (1) with the following data:

$$A=\begin{bmatrix}0&1&-1\\-1&0&0\\1&0&0\end{bmatrix},\quad C=\begin{bmatrix}1&0&1\\0&1&1\\0&0&1\end{bmatrix}$$

$$v(t)=2\pi\begin{bmatrix}-\sin(2\pi t)\\\cos(2\pi t)\\0\end{bmatrix},\quad x(0)=\begin{bmatrix}1\\1\\2\end{bmatrix}.$$

Then, the perspective observation $y(t)$ is given in the form

$$y(t)=\begin{bmatrix}\dfrac{x_1(t)+x_3(t)}{x_3(t)} & \dfrac{x_2(t)+x_3(t)}{x_3(t)}\end{bmatrix}^{\mathrm{T}},$$

and it is not very difficult to check that

(i) $A$ is Lyapunov stable. In fact, $\sigma_-(A)=\phi$ and $\sigma_0(A)=\{0,i,-i\}$ and hence $W_0=\boldsymbol{C}^n$, $W_-=\{0\}$ with $\pi_-=0$ and $\pi_0=I_3$.
(ii) $(C,A)$ is observable, and hence it is detectable, and
(iii) All the other conditions except Assumption 2.1(iii) are satisfied.

Further the data used for our nonlinear observer $\hat{x}(t)$ of the form (26), (27) are given by

$$\hat{x}_0=[2\quad 2\quad 3]^{\mathrm{T}},\quad P^{-1}=\mathrm{diag}\{8.2,8.2,8.2\}=8.2I_3,$$

where the free parameter $P$ is computed so as to satisfy the Lyapunov inequality (28), which, in this example, can be set as $A^*P+PA=0$.

The time evolutions of each component of the estimation error $\rho(t)=x(t)-\hat{x}(t)$ are shown in Fig. 1. The results seem to show that the proposed nonlinear observer works well.

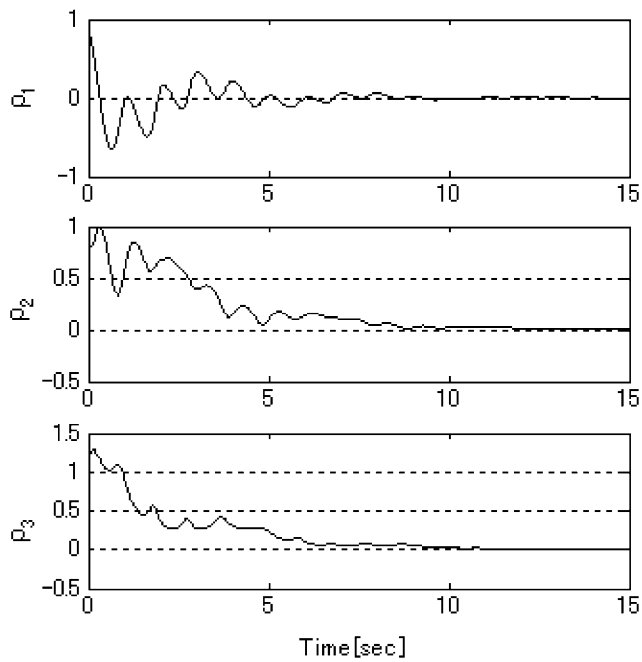


Fig. 1. Estimation error $\rho(t)$.

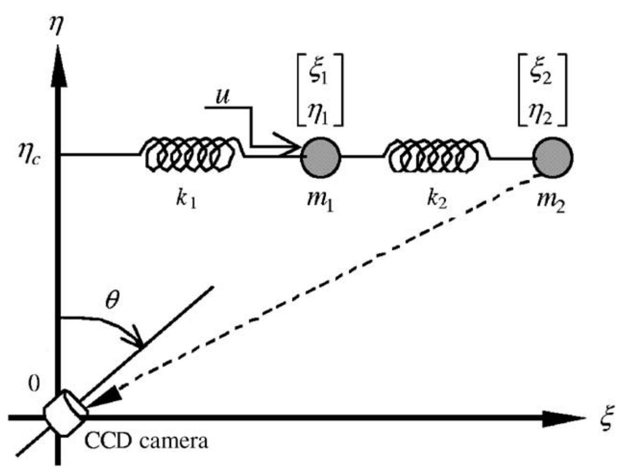


Fig. 2. A two-degree-of-freedom system.

Finally, we should mention that although inequality (15) was not checked in advance, but the fact that the estimation error converges to zero in the computer simulation seems to imply that system (1) is detectable along the trajectory generated by the given input and initial state.

**Example 2.** The second example we consider is a two degree-of-freedom system, consisting of two masses $m_1, m_2$ and two springs with the constants $k_1, k_2$ connected as shown in Fig. 2. The position vectors of masses $m_1, m_2$ are represented by $[\xi_1\ \ \eta_1]^{\mathrm{T}}$, $[\xi_2\ \ \eta_2]^{\mathrm{T}}$, respectively. Assume that the motion takes place only in the $\xi$-direction with $\eta_1 = \eta_2 \equiv \eta_c$, and only the motion of mass $m_2$ is observed via the CCD camera whose optical axis rotated by $\theta$ radian form $\eta$-axis as shown in the figure.

Now, assuming that an external force $u(t) = q\sin\omega t$ is applied to the mass $m_1$ and introducing the state variables

$$x_1 := \xi_1,\ x_2 := \dot{\xi}_1,\ x_3 := \xi_2,\ x_4 := \dot{\xi}_2,$$
$$x_5 := \eta_1 = \eta_2 \equiv \eta_c,$$

the perspective linear system is easily described as

$$\dot{x}(t) = Ax(t) + v(t), \quad x(0) = \bar{x} \in \boldsymbol{R}^5$$
$$y(t) = h(Cx(t)),$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\dfrac{k_1+k_2}{m_1} & 0 & \dfrac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \dfrac{k_2}{m_2} & 0 & -\dfrac{k_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad v(t) = \begin{bmatrix} 0 \\ \dfrac{k_1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t),$$

$$C = \begin{bmatrix} 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & \sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$y = h(Cx) = \frac{x_3\cos\theta - x_5\sin\theta}{x_3\sin\theta + x_5\cos\theta}.$$

Then it is not difficult to check that all the conditions in Assumption 2.1 except inequality (15) are satisfied, and that $A$ is Lyapunov stable with $\sigma_-(A) = \phi$ and hence $\pi_-^*\pi_- = 0$ so that the Lyapunov inequality (28) becomes

$$A^*P + PA \leqslant -a\pi_-^*\pi_- = 0.$$

Next, the numerical values for simulation are set as follows: For the perspective system,

$$m_1 = 2,\ m_2 = 1,\ k_1 = 2,\ k_2 = 1,\ \theta = 0.5[\text{rad}],$$
$$\omega = 1[\text{rad}],\ q = 1,$$
$$x(0) = [0.1 \quad 0.2 \quad 0.3 \quad 0.1 \quad 0.5]^{\mathrm{T}}$$

and for the observer,

$$\hat{x}(0) = [5 \quad 5 \quad 5 \quad 5 \quad 5]^{\mathrm{T}}$$

$$P^{-1} = \begin{bmatrix} 1.1665 & -0.0000 & -0.6905 & 0.0000 & 0 \\ -0.0000 & 2.0950 & -0.0000 & -1.8571 & 0 \\ -0.6905 & -0.0000 & 1.6425 & -0.0000 & 0 \\ 0.0000 & -1.8571 & -0.0000 & 2.3330 & 0 \\ 0 & 0 & 0 & 0 & 0.2562 \end{bmatrix},$$

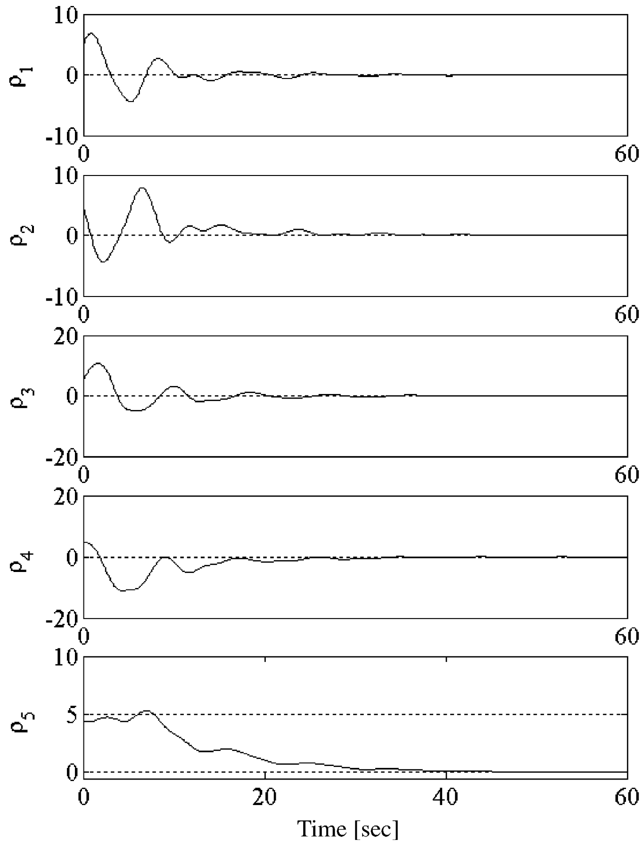


Fig. 3. Estimation errors $\rho(t)$.

where $P$ is a suitably chosen solution of the matrix equation $A^*P + PA = 0$. The time evolutions of each component of the estimation error $\rho(t) = x(t) - \hat{x}(t)$ are depicted in Fig. 3, and the results show that the observer works well.

It should be noted that theoretically the matrix $P$ above can be replaced with any matrix of the form $\mu P$ with $\mu > 0$ to obtain a different convergence property of the estimation error, and further that it can be also replaced with any solution of the Lyapunov inequality $A^*P + PA \leqslant 0$. However, various numerical simulation results with different $\mu > 0$ show that either case $\mu \gg 1$ or $0 < \mu \ll 1$ does not seem to provide a nicer convergence property. Thus more details of this point should be studied as a future problem.

## 5. Concluding remarks

This paper studied a nonlinear observer for perspective linear time-invariant systems arising in machine vision. First a Luenberger-type nonlinear observer was proposed, and then under some reasonable assumptions on a given perspective system, it was shown that it is possible to construct such a Luenberger-type nonlinear observer whose estimation error converges exponentially to zero. Further, to illustrate effectiveness of the proposed observer, some computer simulations using two simple examples were performed, and the results seem to indicate that the proposed nonlinear observer works well.

There are several future problems to be studied. First, although it is obvious that Assumption 2.1(iii) is a sufficient condition for system (1) to be detectable along the trajectory, it may exist a weaker condition than Assumption 2.1(iii) for its detectability. This should be examined as a future problem. Another important problem to be studied is the sensitivity of the proposed observer to noisy observation. The sensitivity is clearly related to a free parameter $P > 0$ to determine an observer gain matrix of (27). Finally, an extension of the present work to time-varying systems is also an interesting problem to be studied.

## Acknowledgements

This work was supported in part by the Japanese Ministry of Education, Science, Sports and Culture under both the Grant-Aid of General Scientific Research C-15560387 and the 21st Century COE (Center of Excellence) Research.

## References

Abdursul, R., Inaba H., & Ghosh, B. K. (2001). Luenberger-type observers for perspective linear systems. *Proceedings of the European control conference*, *CD-ROM*, Porto, Portugal.

Curtain, R. F., & Zwart, H. J. (1995). *An introduction to infinite-dimensional linear systems theory*. Berlin: Springer.

Dayawansa, W. P., Ghosh, B. K., Martin, C., & Wang, X. (1993). A necessary and sufficient condition for the perspective observability problem. *Systems and Control Letters*, *25*, 159–166.

Ghosh, B. K., Inaba, H., & Takahashi, S. (2000). Identification of Riccati dynamics under perspective and orthographic observations. *IEEE Transactions on Automatic Control*, *45*, 1267–1278.

Ghosh, B. K., Jankovic, M., & Wu, Y. T. (1994). Perspective problems in system theory and its applications to machine vision. *Journal of Mathematical Systems, Estimation and Control*, *4*, 3–38.

Ghosh, B. K., & Loucks, E. P. (1995). A perspective theory for motion and shape estimation in machine vision. *SIAM Journal on Control and Optimization*, *35*, 1530–1559.

Ghosh, B. K., & Rosenthal, J. (1995). A generalized Popov-Belevitch–Hautus test of observability. *IEEE Transactions on Automatic Control*, *40*, 176–180.

Inaba, H., Yoshida, A., Abdursul, R., & Ghosh, B. K. (2000). Observability of perspective dynamical systems. *IEEE proceedings of conference on decision and control*, Sydney, Australia. (pp. 5157–5162).

Jankovic, M., & Ghosh, B. K. (1995). Visually guided ranging from observations of points, lines and curves via an identifier based nonlinear observer. *Systems and Control Letters*, *25*, 63–73.

Matveev, A., Hu, X., Frezza, R., & Rehbinder, H. (2000). Observers for systems with implicit output. *IEEE Transactions on Automatic Control*, *45*, 168–173.

Soatto, S., Frezza, R., & Perona, P. (1996). Motion estimation via dynamic vision. *IEEE Transactions on Automatic Control*, *41*, 393–413.

**Rixat Abdursul** was born in Xinjiang, China. He received B.S. degree in Mathematics in 1984 from Xinjiang University and M.S. degree in Information Sciences in 1993 form Tokyo Denki University, and he finished his doctoral course work in 1996 at Tokyo Denki University. From 1984–1993, he served as an instructor in the Department of Mathematics, Xinjiang University, and from 1996–1998 as a research associate in the Department of Information Sciences, Tokyo Denki University. Since 1999, he has been Instructor in the Department of Information Sciences, Tokyo Denki University. He is a member of the Society of Instrument and Control Engineers of Japan and IEEE.

**Hiroshi Inaba** He received the Ph.D. degree from the University of Texas at Austin in 1972. From 1974 to 1976 he served as an Assistant Professor in the Department of Aerospace Engineering, the University of Texas at Austin. In 1977 he joined as Professor in the Department of Mathematical Sciences, Tokyo Denki University (TDU), and since 1986 he has been with the Department of Information Sciences in TDU. From 1993 to 1997, he served as Dean of the Graduate School of Science and Engineering, and since 2000 he has been serving as Director of the Research Institute for Science and Technology, both in TDU. His main research interest is in mathematical systems theory, including the theories of infinite-dimensional linear systems and linear systems over rings. Recently, he has been working on various problems in neural networks and machine vision. Dr. Inaba is a member of the Institute of Systems, Control and Information Engineers of Japan, the Society of Instrument and Control Engineers of Japan, the International Linear Algebra Society (ILAS), and SIAM.

**Bijoy K. Ghosh** (S'78-M'79-SM'90-F'00) received his B.Tech and M.Tech degrees in Electrical and Electronics Engineering from India in 1977 and 1979 respectively. In 1983, he received his Ph.D. in Engineering from the Decision and Control Group of the Division and Applied Sciences at the Harvard University, Cambridge, USA. Since 1983, he has been a faculty member in the Department of Electrical and Systems Engineering at Washington University where he is currently a Professor and directs the center for BioCybernetics and Intelligent Systems. Bijoy's research interests are in Multivariable Control Theory, Machine Vision, Robotics, Biological Control Systems and Systems Biology. In 1988, Bijoy received the American Automatic Control Council's Donald P. Eckman award in recognition of his outstanding contributions in the field of Automatic Control. In 2000, he became a Fellow of the IEEE, for fundamental contributions in Systems Theory with applications to robust control, vision and multisensor fusion. In 1993, he had been an UNDP consultant under the TOKTEN program and visited the Indian Institute of Technology, Kharagpur, India. In 1997, he received the Japan Society for the Promotion of Science Invitation Fellowship for research in Japan and visited Tokyo Denki University and Mechanical Engineering Laboratory, Tsukuba City, Japan. He has also held short term visiting positions at Osaka University, Japan in 1992, Tokyo Institute of Technology, Japan in 1995, Padova University, Italy in 2001 and Institut Mittag-Leffler, Sweden in 2003. Bijoy is a permanent visiting professor at the Tokyo Denki University and in the spring of 2001 he visited the Electrical Engineering Department at Yale University, New Haven, USA. He is presently an elected member of the Board of Governors of the IEEE Control Systems Society and is a committee chair for the technical committee on BioSystems and Control.