

Midterm I

(Make up)

①

① Let A be a 2×2 matrix given by

$$A = \begin{pmatrix} -3 & 1 \\ 0 & -3 \end{pmatrix}$$

(a) calculate A^{100} .

(b) Evaluate the power series

$$I + A + A^2 + A^3 + \dots$$

(c) calculate

$$I + A + A^2 + \dots + A^{100}$$

②

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -4 & -5 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$c = (0 \ 0 \ 1)$$

②

- (a) Is the above system controllable?
- (b) Is the above system observable?
- (c) Let P be the matrix such that

$$F = P^{-1}AP, \quad g = P^{-1}b, \quad h = cP$$

(i) Find P such that

$$F = A \quad \& \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(ii) Find P such that

$$F = A \quad \& \quad h = (1 \ 0 \ 0)$$

(d) Consider the dynamical system

$$\dot{x} = Ax + bU$$

Can you find a feedback K ^{vector} F

such that $U = FX$ &

$$\dot{x} = (A + bF)x$$

where $A + bF$ has eigenvalues at

$$\lambda = -1, -1, -1.$$

(3)

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③ Let A be the matrix

$$A = \begin{pmatrix} \lambda & 1 & 0 & 1 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$$

① compute the eigenvalues and generalized eigenvectors of A .

② compute P such that

$$P^{-1}AP$$

is in the Jordan canonical form.

Also write down the Jordan canonical form.