H. W. 2

① Find the transition matrix $\phi(t,0)$ for the time varying linear system

$$\dot{x}(t) = f(t)\, A\, x(t)$$

where $f(t)$ is a continuous fn of t,

A is a constant $n \times n$ matrix.

② Calculate the transition matrix for the 2 dimensional system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -2\delta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where δ is a constant.

③ Let A, B, C be 3 $n \times n$ matrices.

show that

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$

(Jacobi Bracket Identity)

Here $[A, B] \triangleq AB - BA$.

④ Ⓐ Let A, B be square matrices that do not commute i.e. $AB \neq BA$. We define

$$[A, B] = AB - BA.$$

Define C:

$$e^C = e^A e^B.$$

We know that $C \neq A + B$.

Baker, Campbell & Hausdorff has shown that

$$C = (A+B) + \frac{1}{2}[A, B] + \frac{1}{12}\left\{[[B,A],A] - [[B,A],B]\right\} + \cdots$$

Can you verify the expansion of C upto the 1st two terms. i.e.

$$C = (A+B) + \frac{1}{2}[A, B] + \cdots$$

(4)(B)

Define D:

$$e^{D(t)} = e^{At} e^{Bt} e^{-At} e^{-Bt}$$

Can you calculate D using results that you have already obtained in (4)(A).

(5) Consider the following time varying autonomous system:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 2\cos 7t \\ -3 & 0 & -2\sin 7t \\ -2\cos 7t & 2\sin 7t & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

Find the state transition matrix $\phi(t,0)$

Hint: I have never done this problem.

6 Discuss the controllability properties of the state space equation on page 29 Lec 3 — the equation for "spread of an epidemic disease"