Homework 7

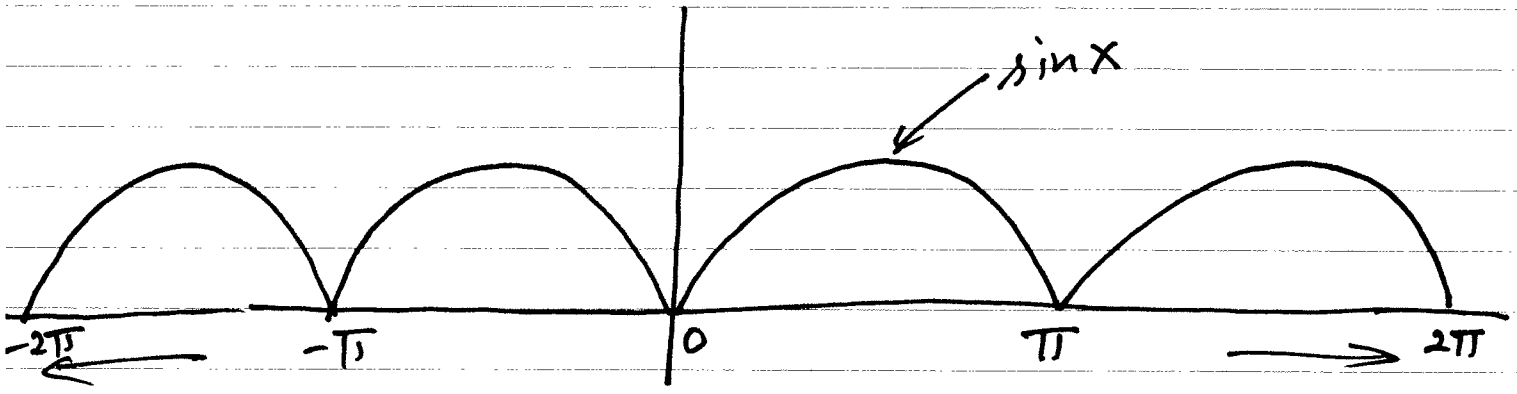
① Calculate the Fourier Series expansion of the function

$$f(x) = |\sin x|$$

Hint:

$$f(x) = \sin x \quad 0 < x < \pi$$

$$f(x+\pi) = f(x) \quad \forall x.$$

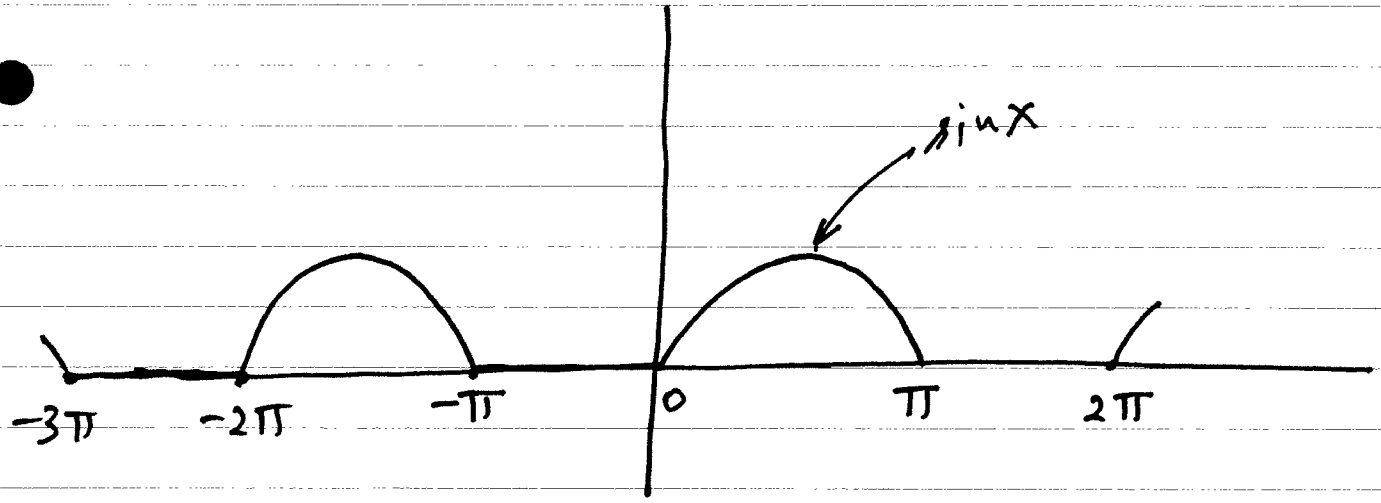


(2) Calculate the Fourier Series expansion of the function.

$$f(x) = \sin x \qquad 0 < x < \pi$$

$$= 0 \qquad \pi < x < 2\pi$$

$$f(x) = f(x + 2\pi) \quad \forall x.$$

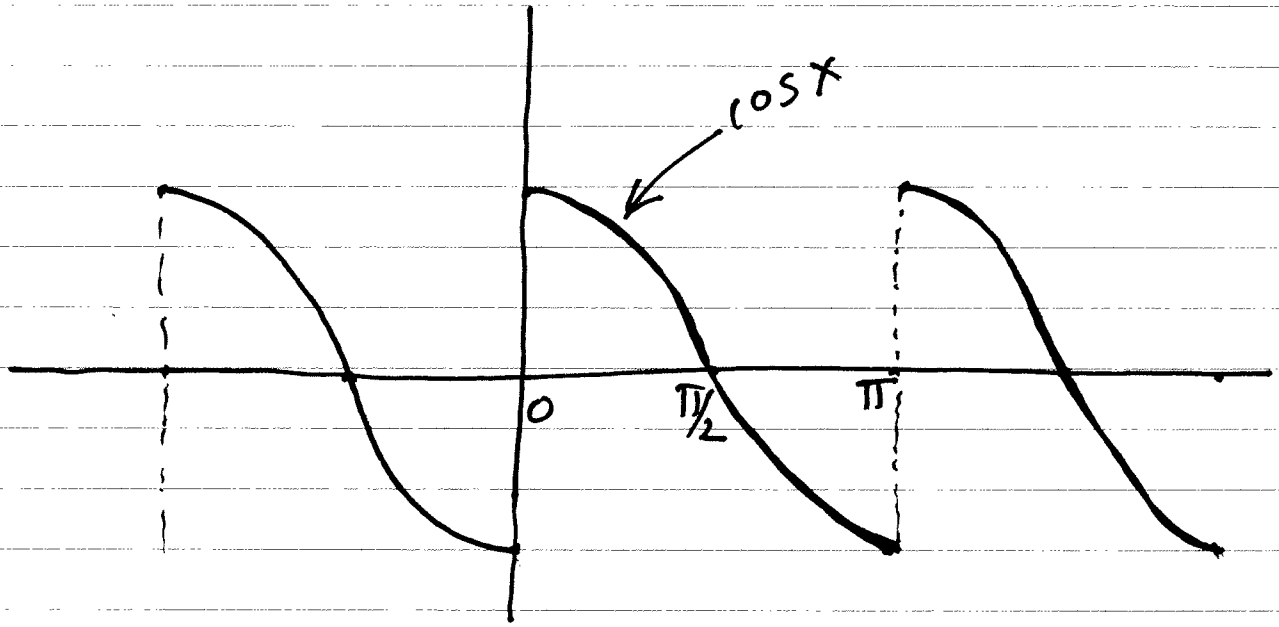


(3) Calculate the Fourier Series expansion of the following function.

$$f(x) = \cos x \quad 0 < x < \pi/2$$

$$= -\cos x \quad -\frac{\pi}{2} < x < 0$$

$$f(x) = f(x+\pi) \quad \forall x$$



(4) (Review)

We have a second order system

$$\ddot{y}(t) + a\dot{y}(t) + b\,y(t) = f(t)$$

$$y(0) = y_0 \;;\; \dot{y}(0) = v_0$$

where a, b are constants. We assume that $a, b, y_0, v_0, f(t)$ are all unknown.

What is known is the solution

$$y(t) = 3e^{-3t} + 4e^{-6t} + 9\sin 10t.$$

Can you find a, b, y_0, v_0 and $f(t)$ in two ways.

(a) Choose $y_p(t) = 3e^{-3t} + 4e^{-6t}$

$y_h(t) = 9\sin 10t.$

(b) Choose $y_p(t) = 9\sin 10t$

$y_h(t) = 3e^{-3t} + 4e^{-6t}$

to solve the problem.

(5) Using properties of Laplace Transform calculate

$$\mathcal{L}\left(t^2 e^{-3t}\cos 5t\right)$$

Hint: you can start from

$$\mathcal{L}\left(\cos 5t\right) = \frac{s}{s^2+25}$$

calculate

$$\mathcal{L}\left(t^2 \cos 5t\right)$$

and finally.

$$\mathcal{L}\left(t^2 e^{-3t}\cos 5t\right).$$

to get the answer.

(6) (A) Calculate $\mathcal{L}^{-1}$ of the following functions.

$$F_1(s) = \frac{5}{s^2+25}$$

$$F_2(s) = \frac{s}{s^2+25}$$

$$F_3(s) = \frac{10s}{(s^2+25)^2}$$

$$F_4(s) = \frac{-25+s^2}{(s^2+25)^2}$$

Hint: $F_3(s) = -F_1'(s)$.

$F_4(s) = -F_2'(s)$.

(B) If $F_5(s) = \dfrac{3s^3+4s^2+5s+6}{(s^2+25)^2}$

Find if possible A, B, C, D such that

- $A F_1(s) + B F_2(s) + C F_3(s) + D F_4(s)$
- $= F_5(s)$.

Hence calculate

$\mathcal{L}^{-1} f_5(s)$

Hint: Use calculator to solve
- equations for A, B, C, D.

(C) If $F_6(s) = \dfrac{3s^3+4s^2+5s+6}{((s+1)^2+25)^2}$

Repeat part (B).

Hint: Try substituting $s_1 = s+1$.
and write $F_6(s_1)$ and proceed.