H. W. 1 Sol^{n}

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(2) order 3 Nonlinear

(4) order 2 "

(6) order 2 Nonlinear

(8) order 2 "

(12) $y = \frac{6}{5} - \frac{6}{5} e^{-20t}$.

$20y = 24 - 24 e^{-20t}$.

$\dot{y} = -\frac{6}{5}(-20) e^{-20t}$.

$= 24 e^{-20t}$.

$\dot{y} + 20y = 24$.

(16) $y' = 25 + y^2$

$y = 5\tan 5x$

$y^2 = 25\tan^2 5x$.

$y' = 5 \times 5\sec^2 5x = 25\sec^2 5x$.

$25 + y^2 = 25(1 + \tan^2 5x) = 25\sec^2 5x$.

$\therefore y' = 25 + y^2$

(24) ??

(34) ??

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(2) $dy = (x+1)^2 dx$

$$y = \int (x+1)^2 dx + c$$

$$y = \frac{(x+1)^3}{3} + c$$

(6) $\frac{dy}{dx} = -2xy^2$

$$\int \frac{dy}{y^2} = -\int 2x \, dx$$

$$-\frac{1}{y} = -x^2 \mp c$$

$$\frac{1}{y} = x^2 + c$$

$$\boxed{y = \frac{1}{x^2 + c}}$$

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$$e^{x} y \frac{dy}{dx} = e^{-y} + e^{-2x} e^{-y}$$

$$e^{x} y \, dy = e^{-y}\left(1 + e^{-2x}\right) dx$$

$$\frac{y\,dy}{e^{-y}} = \frac{1+e^{-2x}}{e^{x}} \, dx.$$

$$\int y e^{y} \, dy = \int \left(e^{-x} + e^{-3x}\right) dx$$

$$y e^{y} - \int e^{y} \, dy = \frac{e^{-x}}{-1} + \frac{e^{-3x}}{-3} + C$$

$$y e^{y} - e^{y} = -e^{-x} - \frac{1}{3} e^{-3x} + C$$

$$\boxed{(y-1) e^{y} = -e^{-x} - \frac{1}{3} e^{-3x} + C}$$

(18)

$$\frac{dN}{dt} + N = Nte^{t+2}$$

$$\frac{dN}{dt} = N\left[te^{t+2} - 1\right]$$

$$\int \frac{dN}{N} = \int \left[te^{t+2} - 1\right] dt.$$

$$\ln N = e^2\left[te^t - \int e^t dt\right] - t + c$$

$$\ln N = e^2\left[te^t - e^t\right] - t + c$$

$$N = e^{\left[e^2\{te^t - e^t\} - t\right]} e^c.$$

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$$\frac{dy}{dx} = \frac{y(x+2) - (x+2)}{y(x-3) + (x-3)}.$$

$$= \frac{(y-1)(x+2)}{(y+1)(x-3)}.$$

$$\int \frac{y+1}{y-1}\, dy = \int \frac{x+2}{x-3}\, dx.$$

$$\Rightarrow \int \frac{y-1+2}{y-1}\, dy = \int \frac{x-3+5}{x-3}\, dx.$$

$$\Rightarrow \int \left[1 + \frac{2}{y-1}\right] dy = \int \left[1 + \frac{5}{x-3}\right] dx.$$

$$\Rightarrow \boxed{y + 2\ln(y-1) = x + 5\ln(x-3) + C}.$$

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$$\frac{dy}{dt} + 2y = 1.$$

$$\frac{dy}{dt} = 1 - 2y$$

$$\frac{dy}{1-2y} = dt$$

$$-\frac{1}{2}\ln(1-2y) = t + c$$

$$\ln(1-2y) = -2t + c_1.$$

$$1 - 2y = e^{-2t} k.$$

$$2y = 1 - k e^{-2t}.$$

$$\boxed{y = \frac{1}{2} - \frac{k}{2} e^{-2t}}$$

$$y(0) = \frac{1}{2} - \frac{k}{2} = \frac{5}{2} \Rightarrow \cancel{\frac{k}{2} = \frac{1}{2} - \frac{5}{2}}$$

$$= -2.$$

$$k = -4.$$

$$y(0) = \frac{1}{2} - \frac{k}{2} = \frac{5}{2}.$$

$$-\frac{k}{2} = \frac{4}{2} \qquad k = -4$$

$$\boxed{y(t) = \frac{1}{2} + 2e^{-2t}.}$$

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(2) $$\frac{dy}{dx} + 2y = 0$$

$$p(x) = 2 \quad f(x) = 0.$$

$$e^{\int p(x)dx} = e^{\int 2dx} = e^{2x}$$

$$y = ce^{-\int p\,dx} = ce^{-2x}$$

(8) $$\frac{dy}{dx} - 2y = x^2 + 5.$$

$$p(x) = -2.$$

$$e^{-\int p(x)dx} = e^{2x}$$

$$y = ce^{2x} + e^{2x}\int e^{-2x}(x^2+5)\,dx$$

$$\int x^2 e^{-2x}\,dx$$

$$= x^2 \frac{e^{-2x}}{-2} - \int \cancel{2}x \frac{e^{-2x}}{-\cancel{2}}\,dx$$

$$= -\frac{1}{2}x^2 e^{-2x} + \int x e^{-2x}\,dx$$

$$= -\frac{1}{2}x^2 e^{-2x} + \frac{x e^{-2x}}{-2} + \int \frac{e^{-2x}}{2}\,dx$$

$$= -\frac{1}{2}x^2 e^{-2x} - \frac{1}{2}x e^{-2x} - \frac{1}{4}e^{-2x}.$$

$$\int 5e^{-2x}\,dx = -\frac{5}{2}e^{-2x}.$$

$$y(x) = ce^{2x} + \cancel{e^{2x}}\left[\cancel{e^{-2x}}\left\{-\frac{x^2}{2} - \frac{x}{2} - \frac{1}{4} - \frac{5}{2}\right\}\right]$$

$$\boxed{y(x) = ce^{2x} - \left[\frac{x^2}{2} + \frac{x}{2} + \frac{11}{4}\right]}$$

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$$(1+x)\frac{dy}{dx} - xy = x(1+x)$$

$$\frac{dy}{dx} - \frac{x}{1+x}y = x$$

$$P(x) = -\frac{x}{1+x} \; ; \; f(x) = x$$

$$\int P(\sigma)\,d\sigma = \int -\frac{x+1-1}{x+1}\,dx$$

$$= \int -\left[1 - \frac{1}{x+1}\right] dx$$

$$= \int -1 + \frac{1}{x+1}\,dx$$

$$= -x + \ln(x+1)$$

$$y_c = c\,e^{[x - \ln(1+x)]}$$

$$= c\,e^{x}/1+x = c\,\frac{e^x}{x+1}.$$

$$e^{\int p(\sigma)\,d\sigma}$$

$$= e^{-x+\ln(1+x)}$$

$$= e^{-x}(1+x)$$

$$\therefore\ y = \frac{c\,e^{x}}{x+1} + \frac{e^{x}}{x+1}\int (x+1)e^{-x}\,x\,dx$$

$$\int (x^2+x)e^{-x}\,dx = e^{-x}\left[-x^2-3x-3\right]$$

$$\int xe^{-x}\,dx = \frac{x\,e^{-x}}{-1} + \int e^{-x}\,dx$$

$$= -xe^{-x} - e^{-x}$$

$$\int x^2e^{-x}\,dx = -x^2e^{-x} + \int 2xe^{-x}\,dx$$

$$= -x^2e^{-x} - 2\left[xe^{-x} + e^{-x}\right]$$

$$= e^{-x}\left[-x^2-2x-2\right]$$

$$y = c\frac{e^x}{x+1} + \frac{\cancel{e^x}\,\cancel{e^{-x}}\left[-x^2-3x-3\right]}{x+1}.$$

$$\boxed{y = c\frac{e^x}{x+1} - \frac{x^2+3x+3}{x+1}.}$$

(16)

$$y\,dx = (y e^{y} - 2x)\,dy$$

$$\frac{dy}{dx} = \frac{y}{ye^{y} - 2x}.$$

$$\frac{dx}{dy} = \frac{ye^{y} - 2x}{y} = e^{y} - \frac{2x}{y}$$

$$\frac{dx}{dy} + \frac{2}{y}x = e^{y}$$

$$p(y) = \frac{2}{y}.$$

$$\int p(\sigma)\,d\sigma = \int \frac{2}{\sigma}\,d\sigma = 2\ln\sigma = \ln(\sigma^{2})$$

$$e^{\int p(\sigma)\,d\sigma} = \sigma^{2}.$$

$$\int -p(\sigma)\,d\sigma = \int -\frac{2}{\sigma}\,d\sigma = -2\ln\sigma = \ln\left(\frac{1}{\sigma^{2}}\right)$$

$$x = \frac{c}{y^2} + \frac{1}{y^2}\int y^2 e^y \, dy .$$

$$\int y^2 e^y dy = y^2 e^y - \int 2y e^y dy .$$

$$= y^2 e^y - 2\left[y e^y - \int e^y dy \right]$$

$$= y^2 e^y - 2y e^y + 2 e^y$$

$$= (y^2 - 2y + 2) e^y .$$

$$\therefore x = \frac{c}{y^2} + e^y \, \frac{y^2 - 2y + 2}{y^2} .$$

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$$\frac{dy}{dx} + y = f(x)$$

$$p(x) = 1$$

$$y_c = c e^{-\int dx} = c e^{-x}.$$

$$y = c e^{-x} + e^{-x} \int e^{x} f(x)\, dx$$

$$\cancel{\therefore\ y(0) = 1 \text{ we have } c = 1.}$$

$$\cancel{y = e^{-x} + e^{-x} \int_0^{x} e^{\sigma} f(\sigma)\, d\sigma.}$$

When $f(x) = 1$

$$y(x) = c e^{-x} + e^{-x} \int e^{x} dx$$

$$= c e^{-x} + e^{-x} e^{x} = 1 + c e^{-x}.$$

when

$f(x) = -1$ we have

$y(x) = -1 + de^{-x}$.

$\because y(0) = 1$ we have

$1 = y(0) = 1 + c \Rightarrow c = 0$.

$\therefore y(x) = 1$ for $0 \le x \le 1$

$\therefore y(1) = 1$

when $x > 1$

$y(x) = -1 + de^{-x}$

$1 = y(1) = -1 + de^{-1}$.

$d = 2e$

$\therefore y(x) = -1 + 2e e^{-x}$

$y(x) = 1 \quad 0 \le x \le 1$

$y(x) = -1 + 2e e^{-x} \quad x \ge 1$.