Midterm II
(Make up)

- Please box your final answer.
- Use bigger fonts.
- Print your name

(1) Consider the o.d.e

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy}{dt} + 2\,y(t) = e^{-3t}$$

$$y(0) = \dot{y}(0) = 0$$

(a) Calculate the particular solution $y_p(t)$? (10 pts)

(b) Calculate the homogeneous solution $y_h(t)$ with arbitrary co-efficients? (10 pts)

(c) write $y(t) = y_h + y_p$ and calculate the (10 pts) unknown co-efficients from the initial conditions

(d) If $\mathcal{L}(y(t)) = Y(s)$, taking the Laplace Transform of the o.d.e calculate $Y(s)$ as a rational function in s. (10 pts)

(e) Calculate $y(t)$ from $Y(s)$ by first obtaining partial fraction expansion. (10 pts)

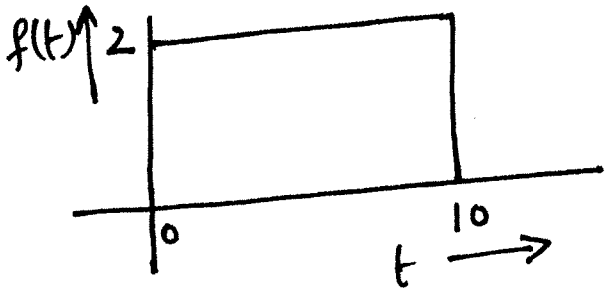
② Calculate

$$\mathcal{L}^{-1}\ \frac{2s+11}{s^2+6s+34} \qquad (25 \text{ points})$$

③

ⓐ Let $f(t)$ be defined as follows:

$$f(t) = 2 \quad 0 \le t \le 10$$
$$= 0 \quad \text{otherwise}$$



Calculate the convolution of $e^{\lambda t}$ and $f(t)$ using the convolution integral (20 pts)

$$\int_0^t e^{\lambda(t-\tau)} f(\tau)\, d\tau.$$

ⓑ We want to solve (10 pts)

$$\ddot{y} + 3\dot{y} + 2y = f(t), \quad y(0) = \dot{y}(0) = 0$$

Write $y(t)$ as a convolution integral. You do not have to solve the integral.