## Mid Semester Exam \#2

## Math 3350: Higher Mathematics for Engineers and Scientists I

## Fall 09 - Section 012

- Time allowed: 1 hour 20 minutes.
- This is an open book exam.
- Answer all questions.
- Show all the necessary work to earn full credit.
- Answers written on the test paper will not be graded.
- Please print your name on the first page of your answer scripts.
- Write your name on all the pages
(1) (a) Using the method of completing the square show that

$$
\mathcal{L}^{-1}\left[\frac{3 s+4}{s^{2}+2 s+2}\right]=e^{-t}(3 \cos t+\sin t) .
$$

(b) Recall the if $\mathcal{L}[f(t)]=F(s)$, then $\mathcal{L}[t f(t)]=-\frac{d F(s)}{d s}$. Use this result to show that

$$
\mathcal{L}\left[t e^{-t}(3 \cos t+\sin t)\right]=\frac{3 s^{2}+8 s+2}{\left(s^{2}+2 s+2\right)^{2}}
$$

(2) Solve for $y(t)$, where $\ddot{y}(t)+5 \dot{y}(t)+6 y(t)=6+2 e^{-t} ; y(0)=\dot{y}(0)=0$.
(3) Consider

$$
\ddot{y}(t)+a \dot{y}(t)+b y(t)=f(t) ; y(0)=y_{0}, \dot{y}(0)=v_{0}
$$

where $a, b, y_{0}$, and $v_{0}$ are unknown constants.
For $f(t)=e^{-2 t}, y(t)$ was recorded as

$$
y(t)=3 e^{-2 t}+2 e^{-t}+e^{-5 t} .
$$

(a) Calculate $a, b, y_{0}$, and $v_{0}$ from this data if possible.
(b) For $f(t)=\sin t$, what should be the corresponding $y(t)$ ?

