Review Problem

① calculate

@a $\displaystyle \int \frac{1}{-3y+10} \, dy$

⓫b $\displaystyle \int \frac{y \, dy}{-3y^2 + 10}$

b $\displaystyle \int e^{13x} \cos 3x \, dx$

↑

use tables in the book

② show that

$$\int e^{ax} \sin bx \, dx = -\frac{1}{b} e^{ax} \cos bx + \frac{a}{b} \int e^{ax} \cos bx \, dx$$

↑

use integration by parts from
Cal-II.

③ write down the answer

$\displaystyle \int e^{10x} \, dx = ?$ $\displaystyle \frac{d}{dx}\left(e^{10x}\right) = ??$

(5) **Discuss what is wrong with the following: and provide the correct solution.**

$$\frac{dy}{dx} = -3y \ ; \quad y(0) = \frac{20}{3}$$

Ans:

$$\frac{dy}{y} = -3 \, dx$$

$$\int \frac{dy}{y} = -\int 3 \, dx$$

$$\ln|y| = -3x + C$$

$$|y| = e^{-3x+C} = e^C e^{-3x} = Ke^{-3x}$$

$$\text{where } K = e^C$$

$$\Rightarrow y = \pm Ke^{-3x}$$

$$\because y(0) = \frac{20}{3} \quad \text{we have} \quad 0 = \pm Ke^{-3\frac{20}{3}}$$

$$\Rightarrow 0 = \pm Ke^{-20}.$$

$$\therefore K = 0 \Rightarrow y = 0e^{-3x}$$

$$= 0.$$

④ Discuss what is wrong with the following:

$$\frac{dy}{dx} + \frac{x+\sin y}{x\cos y + y^3} = 0 \quad , \quad y(0) = 2$$

$$\Rightarrow (x+\sin y)\, dx + (x\cos y + y^3)\, dy = 0$$

$$\Rightarrow \int(x+\sin y)\, dx = -\int(x\cos y + y^3)\, dy$$

$$\Rightarrow \frac{x^2}{2} + x\sin y = -x\sin y - \frac{y^4}{4} + C$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^4}{4} + 2x\sin y = C$$

$$\Rightarrow \frac{(2)^4}{4} = C \qquad \Rightarrow C = 4$$

$$\therefore \boxed{\frac{x^2}{2} + \frac{y^4}{4} + 2x\sin y = 4}$$

(6) Discuss what is wrong with the following:

$$\frac{dy}{dx} = -3y + 4e^{10x} ; \quad y(0) = \frac{1}{13}$$

Ans: $P = 3$ $\qquad Q = 4e^{10x}$.

$$\mu(x) = e^{\int P\,dx} = e^{3x}.$$

$$y(x) = \frac{1}{e^{3x}} \left[\int e^{3x} 4e^{10x}\,dx \right]$$

$$= \frac{1}{e^{3x}} \int 4e^{13x}\,dx .$$

$$= \frac{1}{e^{3x}} \frac{4e^{13x}}{13} = \frac{4}{13} e^{10x} .$$

$$y(x) = \frac{4}{13} e^{10x}$$

$$y(0) = \frac{4}{13} = \frac{1}{13} \quad \leftarrow \text{Does not make any sense.}$$

Can you correct any mistake that you discover.