$\boxed{\text{Home Work}}$ 6

1. Show that

(a) $$\mathcal{L}^{-1}\left(\frac{As+B}{s^2+\omega^2}\right) = A\cos\omega t + \frac{B}{\omega}\sin\omega t$$

(b) $$\mathcal{L}^{-1}\left[\frac{Cs+D}{(s^2+\omega^2)^2}\right] =$$

$$\frac{D}{2\omega^3}\sin\omega t + \frac{C}{2\omega}t\sin\omega t - \frac{D}{2\omega^2}t\cos\omega t$$

2 Calculate inverse Laplace Transforms of

(i) $\dfrac{3s^3}{(s^2+16)^2}$ (ii) $\dfrac{3s^3}{(s^2+16)^2(s+3)}$

(iii) $\dfrac{3s^3}{\left[(s+1)^2+16\right]^2}$ (iv) $\dfrac{3s^3}{\left[(s+1)^2+16\right]^2(s+3)}$

3 Solve the following using Laplace's transform.

(a) $$\frac{dy}{dt} + 3y = te^{-t} \sin 4t.$$

where $y(0) = 0$.

(b) $$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = te^{-t} \sin 4t.$$

where $y(0) = 0$, $y'(0) = 0$.

4 Solve the following 2^{nd} order equation

$$\frac{d^2y}{dt^2} + \omega_0^2 y = \sin \omega t$$

assume $y(0) = 0$
$y'(0) = 0$

where
(a) $\omega \neq \omega_0$
(b) $\omega = \omega_0$.

What happens when ω approaches ω_0.

Remark: This is called the resonance problem, when frequency of the forcing term approaches natural frequency.

5 If $f(t)$ and $g(t)$ are two functions defined on the interval $[0,\infty)$ we define convolution.

$$f * g = \int_0^t f(t-\tau)\, g(\tau)\, d\tau .$$

(i) Show that $f * g = g * f$.

(ii) Convolve the following pairs / functions.

(a) $e^{3t}, e^{4t} \quad t \geq 0$.

(b) $e^{3t}, \sin 5t \quad t \geq 0$.

(c) $t^2, e^{5t} \quad t \geq 0$.

(d) $t, \sin t \quad t \geq 0$.

(iii) For the above pairs of functions verify that

$$\mathcal{L}(f * g) = F(s) \cdot G(s).$$

where

$$\mathcal{L}(f(t)) = F(s)$$

&

$$\mathcal{L}(g(t)) = G(s).$$