Home Work 5

Answers

1

a $F(s) = \int_0^\infty e^{-st} f(t)\, dt$

$$= \int_0^{10} e^{-st} e^{-3t}\, dt$$

$$= \int_0^{10} e^{-(s+3)t}\, dt$$

$$= \left. \frac{e^{-(s+3)t}}{-(s+3)} \right|_0^{10}$$

$$= -\frac{1}{s+3}\left[e^{-(s+3)10} - 1 \right]$$

$$= \frac{1}{s+3}\left[1 - e^{-(s+3)10}\right]$$

(b) $F(s) =$

$$\int_0^{10} e^{-st}\, 1\, dt$$

$$= \left.\frac{e^{-st}}{-s}\right|_0^{10}$$

$$= -\frac{1}{s}\left[e^{-s10} - 1\right]$$

$$= \frac{1}{s}\left[1 - e^{-10s}\right]$$

(c)

$$F(s) = \int_{0}^{10} e^{-st} e^{-9(t-1)}\, dt$$

$$= e^{9} \int_{1}^{10} e^{-(s+9)t}\, dt$$

$$= e^{9} \left. \frac{e^{-(s+9)t}}{-(s+9)} \right|_{1}^{10}$$

$$= -\frac{e^{9}}{s+9} \left[e^{-10(s+9)} - e^{-(s+9)} \right]$$

$$= -\frac{1}{s+9} \left[e^{-10s-90+9} - e^{-s-\cancel{9}+\cancel{9}} \right]$$

$$= \frac{e^{-s} - e^{-10s-81}}{s+9}.$$

(2) Ans:

(a) $$F(s) = \frac{10}{s^2+100}.$$

(b) $$F(s) = \frac{s}{s^2+100}.$$

(c) $$F(s) = \frac{10}{(s+3)^2+100}.$$

$$= \frac{10}{s^2+6s+109}.$$

(d) $$F(s) = \frac{s+3}{(s+3)^2+100}.$$

$$= \frac{s+3}{s^2+6s+109}.$$

3) Ans:

(a) $\mathcal{L}^{-1}\frac{3}{s+5} = 3e^{-5t}$.

(b) $\frac{9}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$.

$\Rightarrow \frac{9}{s+2} = A + \frac{B(s+1)}{(s+2)}$

$\Rightarrow A = \frac{9}{s+2}\Big|_{s=-1} = \frac{9}{1} = 9$

$\frac{9}{s+1} = B + \frac{A(s+2)}{s+1}$.

$\Rightarrow B = \frac{9}{s+1}\Big|_{s=-2} = -9$.

$$\therefore \frac{9}{(s+1)(s+2)} = \frac{9}{s+1} - \frac{9}{s+2}.$$

$$\mathcal{L}^{-1}\left[\ \downarrow\ \right] = 9e^{-t} - 9e^{-2t}.$$

(c) $\mathcal{L}^{-1} \dfrac{s}{s^2+16} = \cos 4t.$

(d) $\mathcal{L}^{-1} \dfrac{5}{s^2+25} = \sin 5t.$

(e) $\mathcal{L}(t^3) = \dfrac{3!}{s^4} = \dfrac{6}{s^4}$

$\mathcal{L}^{-1} \dfrac{18}{s^4} = 3\,\mathcal{L}^{-1}\left(\dfrac{6}{s^4}\right) = 3t^3.$

(f) $\dfrac{s+10}{s^2+100} = \dfrac{s}{s^2+100} + \dfrac{10}{s^2+100}$

$\mathcal{L}^{-1}\left(\dfrac{s+10}{s^2+100}\right) = \mathcal{L}^{-1}\left(\dfrac{s}{s^2+100}\right) + \mathcal{L}^{-1}\left(\dfrac{10}{s^2+100}\right)$

$= \cos 10t + \sin 10t.$

(g) $\dfrac{3s+4}{s^2+8^2} = 3\,\dfrac{s}{s^2+8^2} + \dfrac{1}{2}\,\dfrac{8}{s^2+8^2}$

$\mathcal{L}^{-1}\left(\ \downarrow\ \right) = 3\,\mathcal{L}^{-1}\left(\dfrac{s}{s^2+8^2}\right) + \dfrac{1}{2}\mathcal{L}^{-1}\left(\dfrac{8}{s^2+8^2}\right)$

$= 3\cos 8t + \dfrac{1}{2}\sin 8t.$

(4) Ans.

$$\mathcal{L}^{-1}\frac{s}{s^2+25^2} = \cos 25t.$$

(a) $$\mathcal{L}^{-1}\frac{s-13}{(s-13)^2+25^2} = e^{13t}\cos 25t.$$

(b) $$\mathcal{L}^{-1}\frac{25}{s^2+25^2} = \sin 25t.$$

$$\mathcal{L}^{-1}\frac{25}{(s-13)^2+25^2} = e^{13t}\sin 25t.$$

(c) $$\frac{s+12}{(s-13)^2+25^2} = \frac{(s-13)}{(s-13)^2+25^2} + \frac{25}{(s-13)^2+25^2}$$

$$= e^{13t}\cos 25t + e^{13t}\sin 25t.$$

$$= e^{13t}\left[\cos 25t + \sin 25t\right]$$

(5) Ans:

$$s^2+8s+25$$

$$=(s+4)^2+3^2$$

$$\frac{3s+4}{s^2+8s+25}=$$

$$\frac{3s+4}{(s+4)^2+3^2}=\frac{3(s+4)+4-12}{(s+4)^2+3^2}$$

$$=3\frac{s+4}{(s+4)^2+3^2}-\frac{8}{3}\frac{3}{(s+4)^2+3^2}$$

$$\therefore \mathcal{L}^{-1}(\quad)=3\mathcal{L}^{-1}\frac{s+4}{(s+4)^2+3^2}-\frac{8}{3}\mathcal{L}^{-1}\frac{3}{(s+4)^2+3^2}$$

$$=3e^{-4t}\cos 3t-\frac{8}{3}e^{-4t}\sin 3t.$$

$$= e^{-4t}\left[3\cos 3t - \frac{8}{3}\sin 3t\right]$$

6) Ans:

$$s^2 + 8s - 25 = 0$$

$$s = \frac{-8 \pm \sqrt{64 + 100}}{2}$$

$$= -4 \pm \sqrt{\frac{164}{4}}$$

$$= -4 \pm \sqrt{41}$$

$$s_1 = -4 + \sqrt{41}$$

$$s_2 = -4 - \sqrt{41}$$

$$\frac{3s + 4}{s^2 + 8s - 25} = \frac{3s + 4}{(s - s_1)(s - s_2)}$$

$$= \frac{A}{s - s_1} + \frac{B}{s - s_2}$$

$$\frac{3s+4}{s-s_2} = A + B\,\frac{s-s_1}{s-s_2}.$$

$$A = \left.\frac{3s+4}{s-s_2}\right|_{s=s_1} = \frac{3s_1+4}{s_1-s_2}.$$

$$\frac{3s+4}{s-s_1} = B + A\,\frac{s-s_2}{s-s_1}.$$

$$B = \left.\frac{3s+4}{s-s_1}\right|_{s=s_2} = \frac{3s_2+4}{s_2-s_1}.$$

$$= -A.$$

$$\frac{3s+4}{s^2+8s-25} = A\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

$$\mathcal{L}^{-1}(\downarrow) = A\left(e^{s_1 t} - e^{s_2 t}\right)$$