MATH 2360: Linear Algebra
Fall 2009 – Section 004

Problems on Determinants

1. Evaluate the determinant of the matrix $A = \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$.

2. Using 1., evaluate the determinant of the matrix $B = \begin{bmatrix} 5 & 6 & 10 \\ 4 & 4 & 6 \\ -2 & 3 & 5 \end{bmatrix}$. Clearly indicate your steps.

3. Find all the values of λ which makes the following determinant equal zero:
$$\begin{bmatrix} 3-\lambda & -4 \\ 1 & -2-\lambda \end{bmatrix}$$

4. Compute the determinant, adjoint and use them to compute the inverse for the following matrices.
(a) $A = \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

5. Use Cramer's rule to solve the following systems:
(a)
$$\begin{aligned} x + 2y &= 9 \\ 2x + y &= 6 \end{aligned}$$
(b)
$$\begin{aligned} x + y + z &= 6 \\ 2x + 2y + y &= 9 \\ 3x + y + 2y &= 11 \end{aligned}$$

6. Suppose you are given two $n \times n$ matrices C and D which ha the property that $C = D^T$. Further more it is known that det(CD) = 1. What are the possible values of det(C)?

7. Given that P, Q, R and S are 3×3 matrices with determinants 1, -1, 2 and 4 respectively.
(a) Justify that all the matrices above have a well defined inverse.
(b) Find the values of the following, if possible, if not indicate so:
(i) det(PQRS) (ii) $\det(QR^T)$ (iii) $\det(2QR^{-1})$ (iv) $\det(P + Q^{-1})$ (iv) $\det(R(PQ^{-1})^T)$