Home Work: 7

Due this thursday by 11:00 AM.

(1) Show that each of the following are linear operators on $\mathbb{R}^2$. Describe geometrically what each L.T. accomplishes.

(a) $L(x) = (-x_1, x_2)$

(b) $L(x) = \frac{1}{4} x$

(c) $L(x) = -x$

(d) $L(x) = (x_2, x_1)$

(2) Let L be the linear operator on $\mathbb{R}^2$ where

$$L(x) = (x_1 \cos\alpha - x_2 \sin\alpha,\ x_1 \sin\alpha + x_2 \cos\alpha)^T$$

Describe geometrically the effect of the L.T. (Think rotation).

(3) Let us define a map between $\mathbb{R}^2$ as

$$L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
$$L(x) = x + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Show that L is not a linear operator.

(4) Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined as follows.

$$L\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} ; \quad L\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Calculate $L\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(5) Let $b_1 = (1 \;\; 1 \;\; 0)^T$, $b_2 = (1 \;\; 0 \;\; 1)^T$
$b_3 = (0 \;\; 1 \;\; 1)^T$ and let L be defined as.

$$L(x) = x_1 b_1 + x_2 b_2 + (x_1 + x_2) b_3 .$$

Find a matrix A such that

$$L(x) = Ax.$$