

Home Work 6 .
Prachise for Mid term II
make up .

① Calculate the determinant of the following matrix

$$\begin{pmatrix} 1 & 0 & 5 & 0 \\ 2 & 2 & 3 & 1 \\ 5 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{pmatrix}$$

② Check if the following vectors are l.i.

$$\begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

state your reasons.

③ Show if the following vectors in \mathbb{R}^4 form a basis.

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

④ Consider the following subspace V of \mathbb{R}^4

$$V = \text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \\ 4 \end{pmatrix} \right\}$$

Find a basis of V and calculate its dimension.

⑤ Calculate the co-ordinates of the vector

$$\begin{pmatrix} 3 \\ 7 \\ 9 \end{pmatrix}$$

in \mathbb{R}^3 , with respect to the basis

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

⑥ Describe the null space \mathcal{N} of the matrix A

$$A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 3 & 6 & 3 & 9 \\ 5 & 8 & 3 & 13 \end{pmatrix}, \text{ as span of a set of independent vectors.}$$

What is the dimension of \mathcal{N} ??

⑦ Describe the row space R and the column space C of the matrix A as span of a set of independent vectors.

Verify that

$$\dim R = \dim C \leftarrow \text{This is called the rank of } A.$$

Verify that

$$\dim N + \dim R = \text{Number of columns of } A.$$

\uparrow
This is called the nullity of A .