Solutions HW #1

MATH 2360: Linear Algebra.

§ 1.1

(1) (a) $\left.\begin{aligned} x_1 - 3x_2 &= 2 \\ 2x_2 &= 6 \end{aligned}\right\}$ $x_2 = 3,$ $x_1 = 2+9 = 11$

(c) $\left.\begin{aligned} x_1 + 2x_2 + 2x_3 + x_4 &= 5 \\ 3x_2 + x_3 - 2x_4 &= 1 \\ -x_3 + 2x_4 &= -1 \\ 4x_4 &= 4 \end{aligned}\right\}$

$x_4 = 1$

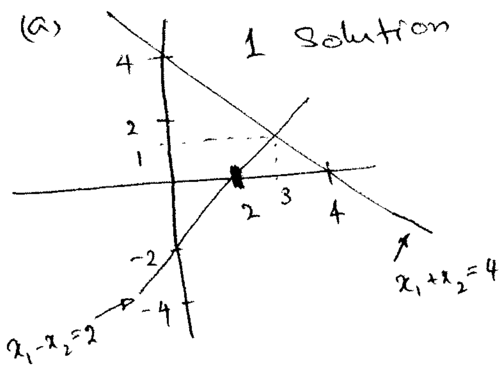
$x_3 = -(-1-2x_4) = -(-1-2) = 3$

$x_2 = \frac{1}{3}(1+2-3) = 0$

$x_1 = 5-1-6-0 = -2$

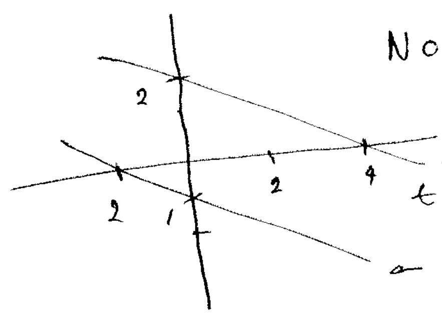
(2) (a) $\begin{pmatrix} 1 & -3 \\ 0 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

(3) (a)



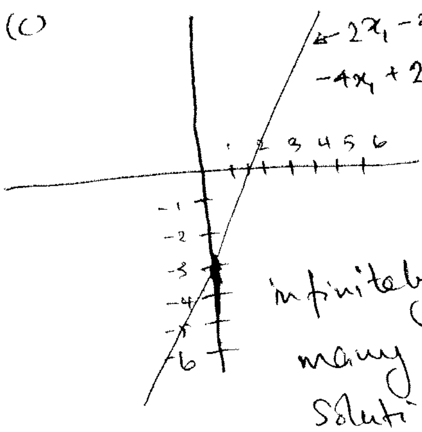
1 Solution

(b)



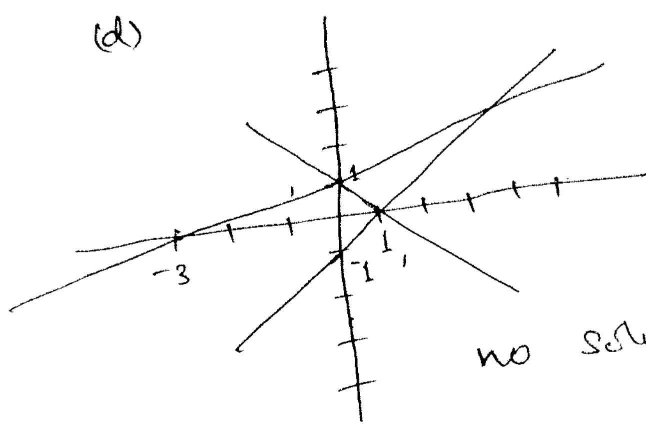
No Solutions. parallel lines

(c)



infinitely many Solutions

(d)



no Solutions.

(4) (c) $\left(\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & -6 \end{array}\right)$ (d) $\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 3 & 3 \end{array}\right)$

⑤ (b) $\left(\begin{array}{ccc|c} 5 & -2 & 1 & 3 \\ 2 & 3 & -4 & 0 \end{array}\right) \Rightarrow \begin{array}{l} 5x_1 - 2x_2 + x_3 = 3 \\ 2x_1 + 3x_2 - 4x_3 = 0 \end{array}$

⑥ (d)
$$x_1 + 2x_2 - x_3 = 1 \quad \text{— } ①$$
$$2x_1 - x_2 + x_3 = 3 \quad \text{— } ②$$
$$-x_1 + 2x_2 + 3x_3 = 7 \quad \text{— } ③$$

Method in Example 3 (p7)

$② - 2 \cdot ① \Rightarrow -5x_2 + 3x_3 = 1$ — ④

$③ + ① \Rightarrow 4x_2 + 2x_3 = 8$ — ⑤

$⑤ + \frac{4}{5}④ \Rightarrow \left(2 + \frac{12}{5}\right)x_3 = 8 + \frac{4}{5}$

$\Rightarrow \frac{22}{5}x_3 = \frac{44}{5}$

$\Rightarrow x_3 = 2$

from ⑤ $\quad x_2 = \frac{1}{4}(8-4) = 1$

from ① $\quad x_1 = 1 - 2 + 2 = 1$

i.e. $x_1 = 1$, $x_2 = 1$, $x_3 = 2$.

Method in Example 4 (p

Augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 3 \\ -1 & 2 & 3 & 7 \end{array}\right) \xrightarrow{R_2 - 2R_1 \to R_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -5 & 3 & 1 \\ -1 & 2 & 3 & 7 \end{array}\right)$$

$$\xrightarrow{R_3 + R_1 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -5 & 3 & 1 \\ 0 & 4 & 2 & 8 \end{array}\right) \xrightarrow{R_3 + \frac{4}{5}R_2 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -5 & 3 & 1 \\ 0 & 0 & 22/5 & 44/5 \end{array}\right)$$

Solving by back substitution we get $x_1 = 1, x_2 = 1, x_3 2$.

⑦ $\left(\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right) \xrightarrow{R_2 - 2R_1 \to R_2} \left(\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{array}\right)$

System 1:

$$\left.\begin{array}{l} 2x_1 + x_2 = 3 \\ \quad\;\; x_2 = -1 \end{array}\right\} \begin{array}{l} x_2 = -1 \\ x_1 = \frac{1}{2}(3+1) = 2 \end{array}$$

$\Rightarrow x_1 = -1 ; x_1 = 2$

System 2:

$$2x_1 + x_2 = -1$$
$$x_2 = 3$$

$\Rightarrow x_2 = 3,$
$x_1 = \frac{1}{2}(-1-3) = -2$

$\Rightarrow x_1 = -2 ; \quad x_3 = 3.$

§1.2

(1)

System	a	b	c	d	e	f	g	h
Row Echelon Form	Yes	No	Yes	Yes	No	No	Yes	Yes
Reduced Row Echelon Form	No	No	Yes	Yes	No	No	Yes	No

(2)

System	a	b	c	d	e	f
Consistent	No	Yes	Yes	Yes	No	Yes
# Solutions	0	1	∞	1	0	1

(3) (a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 3 \\ x_2 = 5 \\ x_3 = -2 \end{array}$

(b) $\left(\begin{array}{ccc|c} 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right)$ Inconsistant $\Rightarrow$ No Solutions

(c) $\left(\begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

* Think of this system interms of the equations.

i.e.

$$x_1 - 3x_2 = 2$$
$$x_3 = -2$$

$\therefore$ Let $x_2 = u$, be the free variable

Then we get

$$x_1 = 2 + 3u$$
$$x_2 = u$$
$$x_3 = -2.$$

§ 1.2

5 (a)

$$x_1 - 2x_2 = 3$$
$$2x_1 - x_2 = 9$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 2 & -1 & 9 \end{array}\right)$$

$$\xrightarrow{R_2 - 2R_1 \to R_2} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 3 & 3 \end{array}\right)$$

$$\xrightarrow{R_3/3 \to R_3} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & 1 \end{array}\right)$$

This part is the Gaussian elimination

This is the Row Echelon Form. Back substitution is equivalent to getting the Reduced Row Echelon form:

$$\xrightarrow{R_1 + 2R_2 \to R_1} \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \quad x_1 = 5$$
$$x_2 = 1$$

(h)

$$x_1 - 2x_2 = 3$$
$$2x_1 + x_2 = 1$$
$$-5x_1 + 8x_2 = 4$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -5 & 8 & 4 \end{array}\right)$$

$$\xrightarrow{R_2 - 2R_1 \to R_2} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 5 & -5 \\ -5 & 8 & 4 \end{array}\right)$$

$$\xrightarrow{R_2/5 \to R_2} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & -1 \\ -5 & 8 & 4 \end{array}\right)$$

$$\xrightarrow{R_3 + 5R_3 \to R_3} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 0 & 2 & 10 \end{array}\right)$$

$$\xrightarrow{R_3/5 \to R_3} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 0 & 1 & 5 \end{array}\right)$$

The system is inconsistant.

(5)(j) $\left.\begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= 1 \\ -x_1 - x_2 + 4x_3 - x_4 &= 6 \\ -2x_1 - 4x_2 + 7x_3 - x_4 &= 1 \end{aligned}\right\}$ $\left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{array}\right)$ (1)

$\xrightarrow{R_2 + R_1 \to R_2}$

$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ -2 & -4 & 7 & -1 & 1 \end{array}\right)$ (2) $\xrightarrow{R_3 + 2R_1 \to R_3}$ $\left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right)$ (3) This is the Row Echelon Form

$\xrightarrow{R_2 - R_1 \to R_2}$

$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow{R_1 + 3R_3 \to R_1} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 4 & 10 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right)$

$\xrightarrow{R_1 - 2R_2 \to R_1}$

$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 2 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \Rightarrow \left.\begin{aligned} x_1 + 6x_4 &= 2 \\ x_2 - x_4 &= 4 \\ x_3 + x_4 &= 3 \end{aligned}\right\}$ let $x_4 = u$, the free variable

$\therefore$ $x_3 = 3 - u$; $x_2 = 4 + u$, $x_1 = 2 - 6u$.

Step (1), (2), (3) are the steps of Gaussian Elimination.

(6) (a) $\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 4 & -3 & 3 \end{array}\right) \xrightarrow{R_2 - 4R_1 \to R_2} \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -7 & 7 \end{array}\right) \xrightarrow{R_2/(-7) \to R_2} \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & 1 & -1 \end{array}\right)$

$\xrightarrow{R_1 - R_2 \to R_1}$ $\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -1 \end{array}\right)$

$\Rightarrow x_1 = 0$; $x_2 = -1$.

(8)

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right) \xrightarrow{R_2+R_1 \to R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 2 & -2 & a & 3 \end{array}\right)$$

$$\xrightarrow{R_3-2R_1 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & -6 & a-2 & 1 \end{array}\right) \xrightarrow{R_2/6 \to R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & -6 & a-2 & 1 \end{array}\right)$$

$$\xrightarrow{R_3+6R_2 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & 0 & a+2 & 4 \end{array}\right)$$

For this system to have a unique solution,

$a \neq -2$.

(11) $x_1 + 2x_2 = 2$ & $x_1 + 2x_2 = 1$

$3x_1 + 7x_2 = 8$ & $3x_1 + 7x_2 = 7$

$$\left(\begin{array}{cc|cc} 1 & 2 & 2 & 1 \\ 3 & 7 & 8 & 7 \end{array}\right)$$

$$\xrightarrow{R_2-3R_1 \to R_2} \left(\begin{array}{cc|cc} 1 & 2 & 2 & 1 \\ 0 & 1 & 2 & 4 \end{array}\right) \xrightarrow{R_1-2R_2 \to R_1} \left(\begin{array}{cc|cc} 1 & 0 & -2 & -7 \\ 0 & 1 & 2 & 4 \end{array}\right)$$

$\therefore$ for system (1) $\left.\begin{array}{l} x_1 + 2x_2 = 2 \\ 3x_1 + 7x_2 = 8 \end{array}\right\}$ $\begin{array}{l} x_1 = -2 \\ x_2 = 2 \end{array}$

for system (2) $\left.\begin{array}{l} x_1 + 2x_2 = 1 \\ 3x_1 + 7x_2 = 7 \end{array}\right\}$ $\begin{array}{l} x_1 = -7 \\ x_2 = 4 \end{array}$

§ 1.3

(1) $A = \begin{pmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{pmatrix}$

$A^T = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 2 \end{pmatrix}$; $B^T \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 2 & 1 & 1 \end{pmatrix}$

(c) $2A - 3B = \begin{pmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 6 \\ -9 & 3 & 3 \\ 6 & -12 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 \\ 5 & -3 & -1 \\ -4 & 16 & 1 \end{pmatrix}$

(d) $(2A)^T - (3B)^T = \begin{pmatrix} 6 & -4 & 2 \\ 2 & 0 & 4 \\ 8 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 3 & -9 & 6 \\ 0 & 3 & -12 \\ 6 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 5 & -4 \\ 2 & -3 & 16 \\ 2 & -1 & 1 \end{pmatrix}$

* Observe that $(2A)^T - (3B)^T = (2A - 3B)^T$!

(2) (a) $\begin{pmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 6+5+4 & 3+15+1 \\ -4+8 & -2+2 \end{pmatrix} = \begin{pmatrix} 15 & 19 \\ 4 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ cannot multiply.

(c) $\begin{pmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 3+4+12 & 2+4+15 \\ 1+16 & 1+20 \\ 8 & 10 \end{pmatrix} = \begin{pmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{pmatrix}$

(d) $\begin{pmatrix} 4 & 6 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{pmatrix} = \begin{pmatrix} 12+24 & 4+6 & 20+36 \\ 6+4 & 2+1 & 10+6 \end{pmatrix} = \begin{pmatrix} 36 & 10 & 56 \\ 10 & 3 & 16 \end{pmatrix}$

(e) $\begin{pmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{pmatrix} =$ cannot multiply.

(f) $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \begin{pmatrix} 3 & 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 4 & 8 & 10 \\ -3 & -2 & -4 & -5 \\ 9 & 6 & 12 & 15 \end{pmatrix}$.

3

(a) $\begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{pmatrix} \Rightarrow$ Results in a 3×3 matrix

(b) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & -4 \\ 6 & -4 \\ 8 & -6 \end{pmatrix} \Rightarrow$ Results in a 1×2 matrix

(c) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow$ Cannot multiply

(d) $\begin{pmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 2 & 1 \end{pmatrix} \Rightarrow$ Cannot multiply

(e) $\begin{pmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{pmatrix}\begin{pmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{pmatrix} \Rightarrow$ cannot multiply

(f) $\begin{pmatrix} 3 & 2 & 4 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \Rightarrow$ Cannot multiply.

4

(a) $\left.\begin{aligned} 3x_1 + 2x_2 &= 1 \\ 2x_1 - 3x_2 &= 5 \end{aligned}\right\} \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

(b) $\left.\begin{aligned} x_1 + x_2 \quad &= 5 \\ 2x_1 + x_2 - x_3 &= 6 \\ 3x_1 - 2x_2 + 2x_3 &= 7 \end{aligned}\right\} \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$

(c) $\left.\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ x_1 - x_2 + 2x_3 &= 2 \\ 3x_1 - 2x_2 - x_3 &= 0 \end{aligned}\right\} \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$

[3.1.3]

(10) $A = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$; $A^2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{4}+\frac{1}{4} & -\frac{1}{2}+\frac{-1}{2} \\ -\frac{1}{2}+\frac{-1}{2} & \frac{1}{2}+\frac{1}{2} \end{pmatrix}$

$$= \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{pmatrix}$$

$A^3 = A^2 \cdot A = A \cdot A^2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{pmatrix} \Rightarrow \ldots A^n = \begin{pmatrix} 1/2 & - \\ -1/2 & \end{pmatrix}$

(13) $A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$; $\mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$; $\mathbf{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $\mathbf{a}_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

(a) $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$

(b) A unique solution exists (i.e. only 1 solution)

(c) Let $\mathbf{c} = \alpha \mathbf{a}_1 + \beta \mathbf{a}_2 \Rightarrow \begin{pmatrix} \alpha \\ \alpha \end{pmatrix} + \begin{pmatrix} 2\beta \\ -2\beta \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \alpha + 2\beta \\ \alpha - 2\beta \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \Rightarrow \begin{matrix} \alpha + 2\beta = -3 \\ \alpha - 2\beta = -2 \end{matrix} \Rightarrow \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -2 & -2 \end{array}\right)$

$\xrightarrow{R_2 - R_1 \to R_2} \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -4 & 1 \end{array}\right) \xrightarrow{R_2/-4 \to R_2} \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & 1 & -1/4 \end{array}\right)$

$\xrightarrow{R_1 - 2R_2 \to R_1} \left(\begin{array}{cc|c} 1 & 0 & -3 + \frac{2}{4} \\ 0 & 1 & -1/4 \end{array}\right) \Rightarrow \beta = \frac{-1}{4}$; $\alpha = \frac{-10}{4} = \frac{-5}{2}$

i.e. $\mathbf{c} = -\frac{5}{2}\mathbf{a}_1 - \frac{1}{4}\mathbf{a}_2$

(14) (a) $A = \left(\underset{a_1}{\begin{pmatrix} 2 \\ -2 \end{pmatrix}} \underset{a_2}{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \right)$; $\underline{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\Rightarrow$ $\underline{b}$ cannot be written as a combination of $\underline{a}_1$ & $\underline{a}_2$

$\Rightarrow$ inconsistant

(b) $A = \left(\underset{a_1}{\begin{pmatrix} 1 \\ 2 \end{pmatrix}} \underset{a_2}{\begin{pmatrix} 4 \\ 3 \end{pmatrix}} \right)$; $\underline{b} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$; $\underline{b} = \underline{a}_1 + \underline{a}_2$

System is consistant

(c) $A = \left(\underset{a_1}{\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}} \underset{a_2}{\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}} \underset{a_3}{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}} \right)$; $\underline{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$; $\underline{b}$ cannot be written in terms of $\underline{a}_1$, $\underline{a}_2$ and $\underline{a}_3$.

$\Rightarrow$ System is inconsistant.

(15) Let $B = \dfrac{1}{d} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$; $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$AB = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{22}/d & -a_{12}/d \\ -a_{21}/d & a_{11}/d \end{pmatrix}$$

$$= \begin{pmatrix} (\underbrace{a_{11}a_{22} - a_{12}a_{21}}_{=d})/d & (\underbrace{-a_{11}a_{12} + a_{12}a_{11}}_{=0})/d \\ (\underbrace{a_{21}a_{22} - a_{22}a_{21}}_{=0})/d & (\underbrace{-a_{21}a_{12} + a_{11}a_{12}}_{=d})/d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$BA = \begin{pmatrix} (\underbrace{a_{22}a_{11} - a_{12}a_{21}}_{=d})/d & (\underbrace{a_{22}a_{12} - a_{12}a_{22}}_{=0})/d \\ (\underbrace{-a_{21}a_{11} + a_{11}a_{21}}_{=0})/d & (\underbrace{-a_{21}a_{12} + a_{11}a_{22}}_{=d})/d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$