

**MATH 1352: CALCULUS II – Section 030**

**MID SEMESTER EXAM I**

1 hour 20 minutes

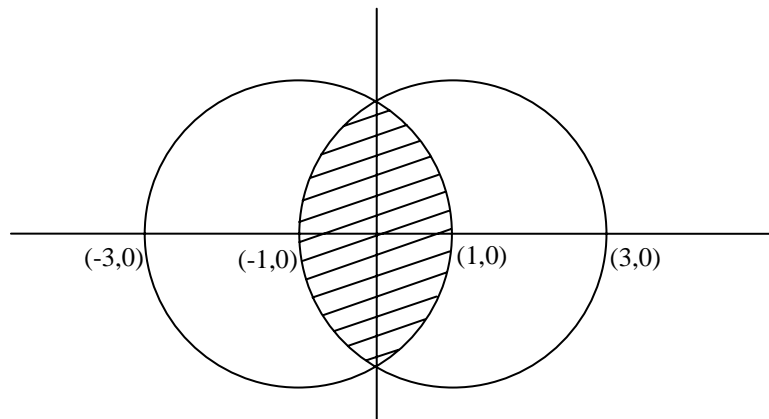
- The use of calculators, textbooks, class notes or mutual consultation is not allowed
- Answers on the question paper will not be accepted.
- Clearly write your name on the answer sheet.

1. Calculate area of intersection between the two circles.

$$(x+1)^2 + y^2 = 4$$

and

$$(x-1)^2 + y^2 = 4.$$



Hints:

i.  $\int \sqrt{4-y^2} dy = 2 \sin^{-1}\left(\frac{y}{2}\right) + \frac{1}{2} y \sqrt{4-y^2}$

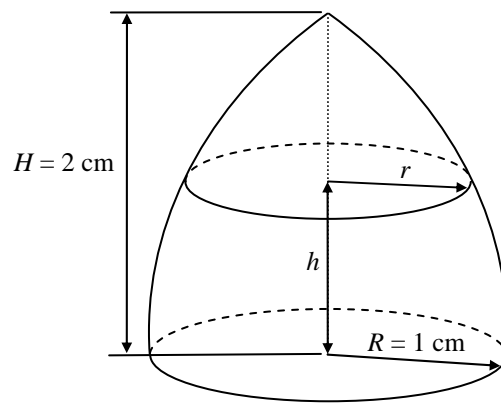
ii. Area =  $2 \int_b^a$  [‘right curve’ – y-axis]

Find ‘a’, ‘b’ and the ‘right curve’

2. We are looking at a solid object (like a tippie) whose base is a circle of radius 1 cm and the height of the object is 2 cm. We are told that every horizontal cross section at height 'h' of the object is a circle of radius 'r', where,

$$r = \left[ 1 - \left( \frac{h}{2} \right)^2 \right] \text{ cm.}$$

Calculate the volume 'V' of the object by writing down the cross sectional volume element,  $\Delta V = \pi r^2 \Delta h$ , and use the definite integral.



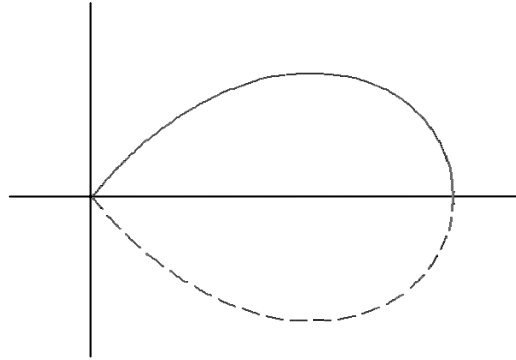
Hint:

$$\text{Volume} = \int_a^b \pi r^2 dh$$

3. Equation of a lemniscate in polar coordinate is given by

$$r^2 = a^2 \cos(2\theta),$$

whose graph is given by the following figure for  $\theta \in [0, \pi/4]$ .



The lemniscate is rotated about the x-axis generating a 3D object. Calculate its surface area using the formula:

$$S = \int_0^{\pi/4} \left[ 2\pi r \sin(\theta) \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \right] d\theta .$$

Hint:

Calculate  $\frac{dr}{d\theta}$  and substitute