Calculus II

Home Work 6

(1) use integration by parts to show that

$$\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

Hint: write

$$\int \cos^n x\,dx = \int \cos^{n-1} x \cdot \cos x\,dx$$

and proceed.

(2) Likewise, show that

$$\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

(3) Calculate

$$\int \tan^{-1} x \, dx$$

Hint: Write $u = \tan^{-1} x$

$$du = \frac{1}{1+x^2} dx$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\int \tan^{-1} x \cdot 1 \, dx =$$

$$\tan^{-1} x \cdot x - \int \frac{1}{1+x^2} x \, dx$$

and proceed.

④ calculate

$$\int e^{ax} \cos bx \, dx = I_1$$

$$\int e^{ax} \sin bx \, dx = I_2$$

⑤ use partial fractions to calculate

(a) $$\int \frac{5x-3}{x^2-2x-3} \, dx$$

(b) $$\int \frac{4-2x}{(x^2+1)(x-1)^2} \, dx$$

(6) Calculate

$$I = \int \frac{dx}{(x^2+4)^2}$$

Hint: Try $x = 2\tan\theta$

$$dx = 2\sec^2\theta\, d\theta$$

$$I = \int \frac{2\sec^2\theta\, d\theta}{(4\tan^2\theta + 4)^2}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{4^2 \sec^4\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta$$

$\sec^2\theta$

and proceed.

(7) Calculate

(a) $\int \frac{3x+2}{x^2+2x+3}\,dx$

(b) $\int \frac{3x+2}{(x^2+2x+3)^2}\,dx$

(8) calculate

(a) $\int \frac{\sin\theta\, d\theta}{\cos^2\theta+\cos\theta-2}$

(b) $\int \frac{x^4}{(x^2+1)^2}\,dx$