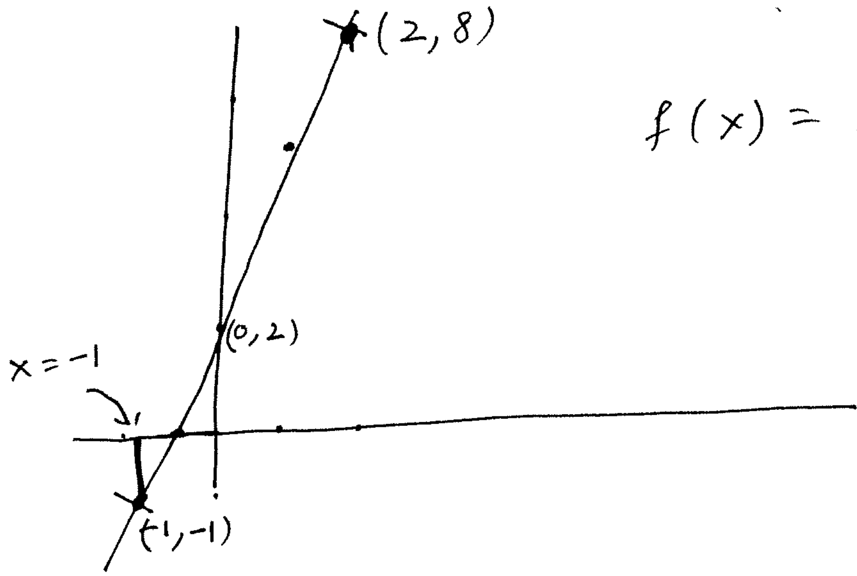
H. W. 3 (Answers)

1



$$f(x) = 3x + 2$$

$$f'(x) = \frac{d}{dx}[3x+2] = 3$$

$$s = \int_{-1}^{2} \sqrt{1 + f'(x)^2}\, dx$$

$$= \int_{-1}^{2} \sqrt{1 + 9}\, dx = \sqrt{10}\, x \Big|_{-1}^{2}$$

$$= \sqrt{10}\,[2 - (-1)]$$

$$= 3\sqrt{10}.$$

4 $f(x) = x^{3/2} \cdot \ x \in [0, 4]$

$f'(x) = \frac{3}{2} x^{1/2}$

$f'(x)^2 = \frac{9}{4} x$

$\sqrt{1 + f'(x)^2} = \sqrt{1 + \frac{9}{4} x}$

$$\frac{d}{dx}\left(1 + \frac{9}{4}x\right)^{3/2} = \frac{3}{2}\left(1 + \frac{9}{4}x\right)^{1/2} \cdot \frac{9}{4}$$

$$= \frac{27}{8}\left(1 + \frac{9}{4}x\right)^{1/2}$$

$$\int \left(1 + \frac{9}{4}x\right)^{1/2} dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2}.$$

$$\therefore \ s = \int_0^4 \left(1 + \frac{9}{4}x\right)^{1/2} dx$$

$$= \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2} \Bigg|_0^4$$

$$= \frac{8}{27}\left[10^{3/2} - 1\right] = \frac{8}{27}\left[10\sqrt{10} - 1\right]$$

$$= 9.0734148$$

7

$$f(x) = \frac{1}{12}x^5 + \frac{1}{5}\frac{1}{x^3} \qquad x \in [1, 2]$$

$$f'(x) = \frac{1}{12}5x^4 + \frac{1}{5}(-3)\frac{1}{x^4}$$

$$= \frac{5}{12}x^4 - \frac{3}{5}(x^{-4})$$

$$f'(x)^2 = \frac{25}{144}x^8 + \frac{9}{25}x^{-8} - \cancel{2}\,\frac{\cancel{5}}{\cancel{12}}\,\frac{\cancel{3}}{\cancel{5}} \quad {}_{\cancel{4}\,2}$$

$$1 + f'(x)^2 = \frac{1}{2} + \frac{25}{144}x^8 + \frac{9}{25}x^{-8}$$

$$= \frac{72 \cdot 25 \cdot x^8 + 25^2 x^{16} + 9 \cdot 144}{144 \cdot 25 \cdot x^8}$$

$$\cancel{(25x^8 + a)^2 = 25^2 x^{16} + 9 \cdot 144 + 72 \cdot 25 x^8}_{\cancel{3 \cdot 12}}$$

$$= \frac{(25x^8 + 36)^2}{(60x^4)^2}$$

$$\sqrt{1 + f'(x)^2} = \frac{25x^8 + 36}{60x^4} = \frac{25}{60}x^4 + \frac{36}{60}x^{-4}$$

$$s = \int_1^2 \sqrt{1 + f'(x)^2}\, dx$$

$$= \int_1^2 \left[\frac{25}{60} x^4 + \frac{36}{60} x^{-4}\right] dx$$

$$= \frac{25}{60}\frac{x^5}{5} + \frac{36}{60}\frac{x^{-3}}{(-3)} \Big|_1^2$$

$$= \frac{\overset{5}{\cancel{25}}}{\cancel{6}0}\frac{x^5}{\cancel{5}} - \frac{12}{60} x^{-3} \Big|_1^2$$

$$= \frac{5}{60} x^5 - \frac{12}{60} x^{-3} \Big|_1^2$$

$$= \left(\frac{5}{60} 64 - \frac{12}{60}\frac{1}{8}\right) - \left(\frac{5}{60} - \frac{12}{60}\right)$$

$$= \frac{5}{60}\left[64 - 1\right] + \frac{12}{60}\left(1 - \frac{1}{8}\right)$$

$3 \cancel{2}\ 2.$
$5 \cdot \cancel{8} \cdot 2 \cdot 2$

$$= \frac{63}{12} + \frac{\cancel{12}}{\underset{5}{\cancel{60}}} \cdot \frac{7}{8} = \frac{63}{12} + \frac{7}{40} = \frac{630 + 21}{120}$$

$$= \frac{651}{120}$$

$$= 5.425$$

10

$$f(x) = (e^{2x}-1)^{1/2} - \sec^{-1}(e^{x})$$

$$x \in [0, \ln 2]$$

$\sec^{-1}(e^{x}) = \cos^{-1}(e^{-x})$ when $e^{x} \geqslant 1$

ie $x \geqslant 0$

$$\frac{d}{dx}\left[\sec^{-1}(e^{x})\right] = \frac{d}{dx}\cos^{-1}(e^{-x})$$

$$= \frac{-1}{\sqrt{1-e^{-2x}}}(-1)e^{-x}$$

$$= \frac{\cancel{e^{-x}}}{\cancel{e^{-x}}\sqrt{e^{2x}-1}} = \frac{1}{\sqrt{e^{2x}-1}}.$$

$$\frac{d}{dx}(e^{2x}-1)^{1/2} = \frac{1}{\cancel{2}}(e^{2x}-1)^{-1/2}\cancel{2}e^{2x}$$

$$= \frac{e^{2x}}{\sqrt{e^{2x}-1}}$$

$$f'(x) = \frac{e^{2x}-1}{\sqrt{e^{2x}-1}} = \sqrt{e^{2x}-1}.$$

$$f'(x)^{2} = e^{2x}-1$$

$$1 + f'(x)^2 = e^{2x}$$

$$\sqrt{1 + f'(x)^2} = e^x$$

$$s = \int_0^{\ln 2} \sqrt{1 + f'(x)^2}\, dx$$

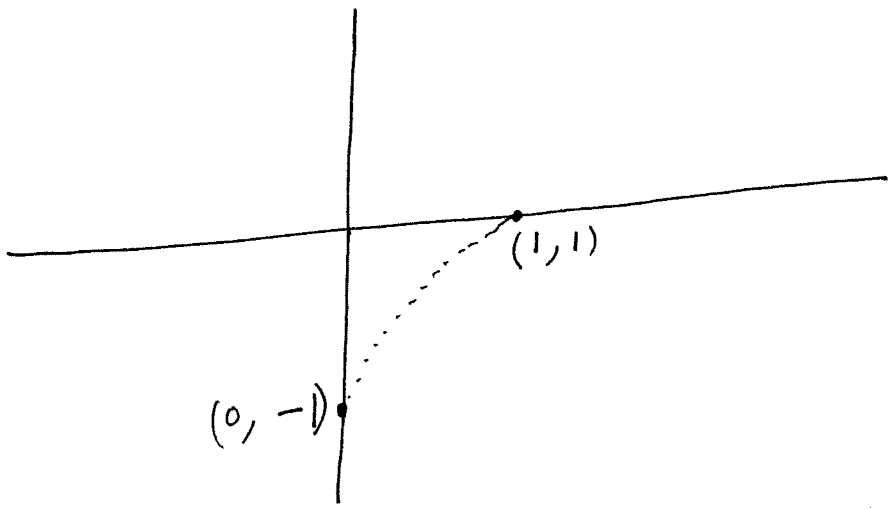
$$= \int_0^{\ln 2} e^x\, dx = e^x \Big|_0^{\ln 2}$$

$$= 2 - 1 = 1.$$

12

$$(y+1)^2 = 4x^3$$

$(0,-1)$ & $(1,1)$



$$(y+1) = +\sqrt{4x^3} = +2x^{3/2}.$$

$$\boxed{y = -1 + 2x^{3/2}}$$

$$\frac{dy}{dx} = 2 \cdot \frac{3}{2} x^{1/2} = 3x^{1/2}$$

$$\left(\frac{dy}{dx}\right)^2 = 9x$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+9x}.$$

$$s = \int_0^1 \sqrt{1+9x}\, dx = \frac{2}{27}(1+9x)^{3/2} \Big|_0^1$$

$$= \frac{2}{27}\left[10\sqrt{10} - 1\right]$$

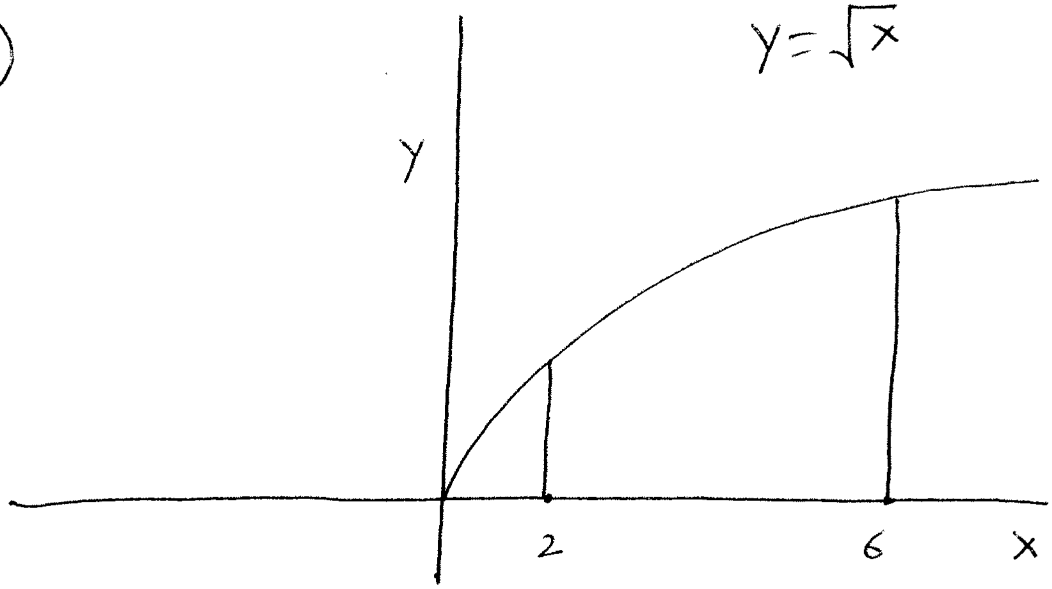
$$\frac{d}{dx}(1+9x)^{3/2}$$

$$= \frac{3}{2}(1+9x)^{1/2} \cdot 9$$

$$= \frac{27}{2}\sqrt{1+9x}$$

2

14



$$S = 2\pi \int_2^6 \sqrt{x}\,\sqrt{1+f'(x)^2}\;dx$$

$$f(x) = x^{1/2}; \quad f'(x) = \frac{1}{2}x^{-1/2}$$

$$[f'(x)]^2 = \frac{1}{4}\frac{1}{x}$$

$$\sqrt{x}\,\sqrt{1+f'(x)^2}$$

$$= \sqrt{x\left(1+\frac{1}{4x}\right)} = \sqrt{\cancel{x}\,\frac{4x+1}{4\cancel{x}}}$$

$$= \frac{\sqrt{4x+1}}{2} = \frac{1}{2}\sqrt{4x+1}$$

$$S = \cancel{2}\pi \frac{1}{\cancel{2}} \int_2^6 \sqrt{4x+1}\, dx$$

$$= \pi \int_2^6 \sqrt{4x+1}\, dx$$

$$\frac{d}{dx}(4x+1)^{3/2} = \frac{3}{\cancel{2}}(4x+1)^{1/2}\,\cancel{4}2 = 6(4x+1)^{1/2}$$

$$\frac{d}{dx}\left[\frac{1}{6}(4x+1)^{3/2}\right] = (4x+1)^{1/2}$$

$$S = \pi\ \frac{1}{6}(4x+1)^{3/2}\Big|_2^6$$

$$S = \frac{49\pi}{3}$$

$$= \frac{\pi}{6}\left[25^{3/2} - 9^{3/2}\right]$$

$$= \frac{\pi}{6}\left[25 \cdot 5 - 9 \cdot 3\right]$$

$$= \frac{\pi}{6}\left[125 - 27\right] = \frac{\pi}{\underset{3}{\cancel{6}}}\ \overset{49}{\cancel{98}} = \frac{49\pi}{3}$$

16 $f(x) = \frac{x^4}{4} + \frac{1}{8}x^{-2}$

$$f'(x) = x^3 + \frac{1}{\cancel{8}_4}(-\cancel{2})x^{-3}$$

$$= x^3 - \frac{1}{4}x^{-3}$$

$$[f'(x)]^2 = x^6 + \frac{1}{16}x^{-6} - \frac{\cancel{2}}{\cancel{4}_2}$$

$$= x^6 + \frac{1}{16}x^{-6} - \frac{1}{2}$$

$$\sqrt{1 + f'(x)^2} = \sqrt{\frac{1}{2} + x^6 + \frac{1}{16}x^{-6}} = \left(x^3 + \frac{1}{4}x^{-3}\right)$$

$$S = 2\pi \int_1^2 \left(\frac{x^4}{4} + \frac{1}{8}x^{-2}\right)\left(x^3 + \frac{1}{4}x^{-3}\right) dx$$

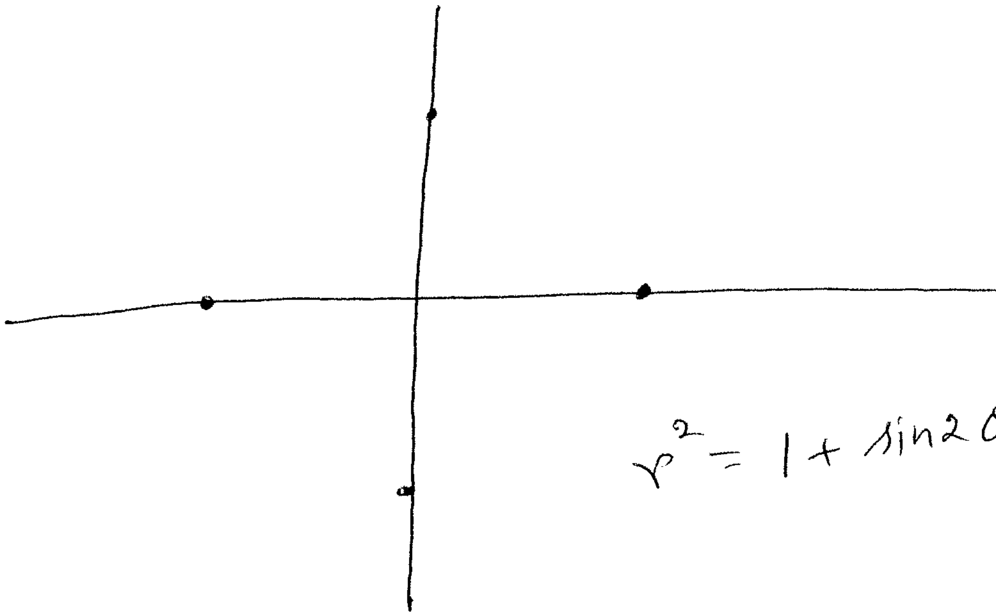
$$= 2\pi \int_1^2 \left[\frac{x^7}{4} + \frac{1}{8}x + \frac{1}{16}x + \frac{1}{32}x^{-5}\right] dx$$

$$= 2\pi \left[\frac{x^8}{32} + \frac{x^2}{16} + \frac{x^2}{32} + \frac{1}{32}\frac{x^{-4}}{(-4)}\right]_1^2$$

$$= 2\pi \left\{ \left[\frac{16\cdot 16}{32_2} + \frac{4}{16} + \frac{4}{32} - \frac{1}{128}\,\frac{1}{16} \right] - \left[\frac{1}{32} + \frac{1}{16} + \frac{1}{32} - \frac{1}{128} \right] \right\}.$$

$$= 2\pi \left\{ \left[8 + \frac{1}{4} + \frac{1}{8} - \frac{1}{16\cdot 128} \right] - \left[\frac{1}{32} + \frac{1}{16} + \frac{1}{32} - \frac{1}{128} \right] \right\}.$$

18 $r = \sin\theta + \cos\theta$.



$r^2 = 1 + \sin 2\theta$

$$\frac{dr}{d\theta} = \cos\theta - \sin\theta$$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{r^2 + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta}.$$

$$= \sqrt{1 + r^2 - \sin 2\theta}$$

$$= \sqrt{2}$$

$\cancel{A} = \cancel{\Phi}$

$$s = \int_0^{2\pi} \sqrt{1 + r^2 - \sin 2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{2}\, d\theta = \sqrt{2}\, 2\pi = 2\sqrt{2}\,\pi$$

20

14

$r = e^{1-\theta}$ $\quad r^2 = e^{2-2\theta}$

$\frac{dr}{d\theta} = e^{1-\theta}(-1)$

$\left(\frac{dr}{d\theta}\right)^2 = e^{2-2\theta}$

$r^2 + \left(\frac{dr}{d\theta}\right)^2 = e^{2-2\theta} + e^{2-2\theta}$

$= 2e^{2-2\theta}$

$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{2}\, e^{1-\theta}$

$\therefore s = \int_0^1 \sqrt{2}\, e^{1-\theta}\, d\theta$

$= \sqrt{2} \left. \frac{e^{1-\theta}}{(-1)} \right|_0^1$

$= -\sqrt{2}\,(e^0 - e^1) = -\sqrt{2}\,(1-e)$

$s = \sqrt{2}\,(e-1)$

(23) $r = 5 \quad 0 \le \theta \le \pi/3$

$$S = 2\pi \int_0^{\pi/3} (5\sin\theta)\sqrt{25 + 0^2}\, d\theta$$

$$= 50\pi \left[-\cos\theta\right]\Big|_0^{\pi/3}$$

$$= 50\pi \left[-\cos\frac{\pi}{3} + 1\right]$$

$$= 50\pi \left[1 - \cos \pi/3\right]$$