SSM 501, Midterm Examination II

Maximum Points: 60, Maximum Time allowed: 90

All problems are worth 10 points. For each of the following questions, show all intermediate calculations. Calculators can be used for arithmatic operations only. Symbolic calculations and matlab will not be allowed. This test is open book and open notes but not open to mutual consultation. Extra time will not be given.

1. Let
$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$
be a 4×4 matrix. Calculate the characteristic polynomial $p(\lambda) = det(\lambda I - A)$ (10 pts).

2. The matrix
$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$
has eigenvalues at 0, 1, 1 and -1. Compute an eigenvector (5 pts) and a generalized eigenvector (5 pts) for the matrix A corresponding to the eigenvalue 1.

3. A 2×2 matrix A has repeated eigenvalues at $-4, -4$. We want to write
$$A^3 e^{At} = \alpha_0(t) I + \alpha_1(t) A$$
and obtain α_0 and α_1 by solving an equation of the form
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{bmatrix} \alpha_0(t) \\ \alpha_1(t) \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}.$$
What should we choose for a, b, c and d (5 pts)? What should we choose for $f(t)$ and $g(t)$ (5 pts)? Explain your choice with a reason.

4. Find a basis of the row space (5 pts) and null space (5 pts) of the skew symmetric matrix
$$A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}.$$

5. Let P be a plane in $\mathbb{R}^4$ described by the following set of equations
$$3x_1 + 2x_2 + x_3 - 2x_4 = 0$$
$$2x_1 + 3x_2 + 2x_3 - x_4 = 0$$
Let $u = (1 \ \ -1 \ \ 1 \ \ 1)$ be a vector contained in the plane P. Find another vector perpendicular to u contained in the plane P (10 pts).

6. For an arbitrary constant parameter θ, the vectors
$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
are eigenvectors of the 2×2 symmetric matrix
$$W = \begin{pmatrix} cos\theta & sin\theta \\ sin\theta & cos\theta \end{pmatrix}.$$
Calculate the corresponding eigenvalues (5 pts). Also write down the orthogonal matrix P such that $P^T W P$ is diagonal (5 pts).