SSM 501, First Midterm Examination

1. For three distinct real numbers $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$ show if it is true that the three vectors

$$v_1 = (1 \quad 1 \quad 1)$$

$$v_2 = (\lambda_1 \quad \lambda_2 \quad \lambda_3)$$

and

$$v_3 = (\lambda_1^2 \quad \lambda_2^2 \quad \lambda_3^2)$$

are linearly independent. Otherwise argue why they are not.

2. In R^4 let P_1 and P_2 be two planes described parametrically as follows:

$$P_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + t_1 \begin{pmatrix} 3 \\ 4 \\ 6 \\ 8 \end{pmatrix} + t_2 \begin{pmatrix} 5 \\ 9 \\ 4 \\ 0 \end{pmatrix} : t_1, t_2 \in R \right\}$$

$$P_2 = \left\{ \begin{pmatrix} 6 \\ 2 \\ 0 \\ 5 \end{pmatrix} + t_3 \begin{pmatrix} 6 \\ 2 \\ 0 \\ 0 \end{pmatrix} + t_4 \begin{pmatrix} 7 \\ 0 \\ 0 \\ 0 \end{pmatrix} : t_3, t_4 \in R \right\}$$

Calculate the intersection of P_1 and P_2.

3. In R^3 let ℓ_1 and ℓ_2 be two lines described parametrically as follows:

$$\ell_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} : t_1 \in R \right\}$$

$$\ell_2 = \left\{ \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + t_2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} : t_2 \in R \right\}.$$

   (a) Write down the equations of the two lines ℓ_1 and ℓ_2 in cartesian coordinates.

   (b) Show that the two lines ℓ_1 and ℓ_2 do not intersect.

   (c) Let ℓ_2' be another line parallel to ℓ_2 such that ℓ_2' and ℓ_1 intersect at a point. Find equation of the plane that contains the two lines ℓ_2' and ℓ_1.

4. Let V be the set of vectors in a vector space R^4 such that the following two conditions are satisfied.

   - The first coordinate of the vectors in V is negative of the fourth coordinate.
   - The second coordinate of the vectors in V is equal to the third coordinate.

   (a) Calculate the dimension of V by obtaining a basis of V.

   (b) Let u be the vector $(1 \quad 1 \quad -1 \quad 1)$, obtain $\text{proj}_V u$ the projection of the vector u on the space V. Recall that $\text{proj}_V u$ is the vector w such that $u - w$ is perpendicular to the space V.