ESE 501, Final Exam

All problems are of equal weight; Maximum Time Allowed: 120 minutes

For each of the following questions, show all immediate calculations. Calculators are not allowed and will not be required. This test is open book and open notes but not open to mutual consultation. Extra time will not be given.

1. Define

$$A = \begin{pmatrix} -10 & 18 \\ -6 & 11 \end{pmatrix}$$

and consider

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}.$$

Sketch the phase portrait around the point (0,0).

2. Showing all steps, calculate

$$e^{\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} t}.$$

3. Calculate a set of two L.I. eigenvectors/generalized eigenvectors for the matrix

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ -3 & 2 & 2 \end{pmatrix}$$

for the eigenvalue -1.

4. Solve the following ODE

$$\begin{aligned} \dot{x_1} &= -x_1 + x_2 \\ \dot{x_2} &= -2x_2 + e^{-3t} \end{aligned}$$

with initial condition

$$x_1 0 = x_2(0) = 0$$

to calculate $x_1(t)$.

5. Let

$$u = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$$

be a vector in $\mathbf{R}^3$. We define a linear transformation T as follows: $T : \mathbf{R}^3 \mapsto \mathbf{R}^3, \quad V \mapsto proj_u V$.

(a) Describe the null space of T, i.e., tell me if it is a line, plane, affine, or

homogeneous etc.
(b) Calculate a basis of the null space of T.

6. Project the vector

$$\begin{pmatrix} 6 \\ 3 \\ -5 \end{pmatrix}$$

onto the plane spanned by

$$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \quad \& \quad \begin{pmatrix} 1 \\ 9 \\ 15 \end{pmatrix}.$$