ESE501 Home Work Nine

1. Verify by explicit differentiation that

$$x(t) = \int_0^t e^{A(t-\tau)}\ b\ u(\tau)\ d\tau$$

would satisfy the ordinary differential equation

$$\dot{x}(t) \ = \ A\ x(t) \ + \ b\ u(t),$$

where $x(0) = 0$.

2. Let A be a $n \times n$ matrix with eigenvalues at $\lambda_1,\ \lambda_2,\ \lambda_3,\ \ldots\ ,\ \lambda_n$. Show that

$$det\ A = \ \lambda_1\ \lambda_2\ \lambda_3\ \ldots\ \lambda_n,$$

and

$$trace\ A = \ \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n.$$

3. Let A be a 3×3 matrix with distinct eigenvalues at $\lambda_1,\ \lambda_2,\ \lambda_3$. Define a 6×6 matrix B as follows

$$B = \begin{pmatrix} A & I \\ 0 & A \end{pmatrix}.$$

   (a) If v_1, v_2 and v_3 are three linearly independent eigenvectors of A, show that

$$\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \begin{pmatrix} v_2 \\ 0 \end{pmatrix}, \begin{pmatrix} v_3 \\ 0 \end{pmatrix},$$

   are three linearly independent eigenvectors of B.

   (b) Show that B does not have any other linearly independent eigenvectors. In fact the three other linearly independent generalized eigenvectors are

$$\begin{pmatrix} v_1 \\ v_1 \end{pmatrix}, \begin{pmatrix} v_2 \\ v_2 \end{pmatrix}, \begin{pmatrix} v_3 \\ v_3 \end{pmatrix},$$

4. A 3×3 matrix A has eigenvalues repeated at -2, -2 and -2 and a single chain of generalized eigenvectors at

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

   (a) Calculate e^{At} from this data.

   (b) Can you write down the matrix A from this data?

5. Let A be a 2×2 matrix with eigenvalues repeated at 0.3, 0.3. Calculate

$$\sum_{j=1}^{N} j\ A^{j-1}, \text{ and } \sum_{j=1}^{\infty} j\ A^{j-1},$$

in terms of A.