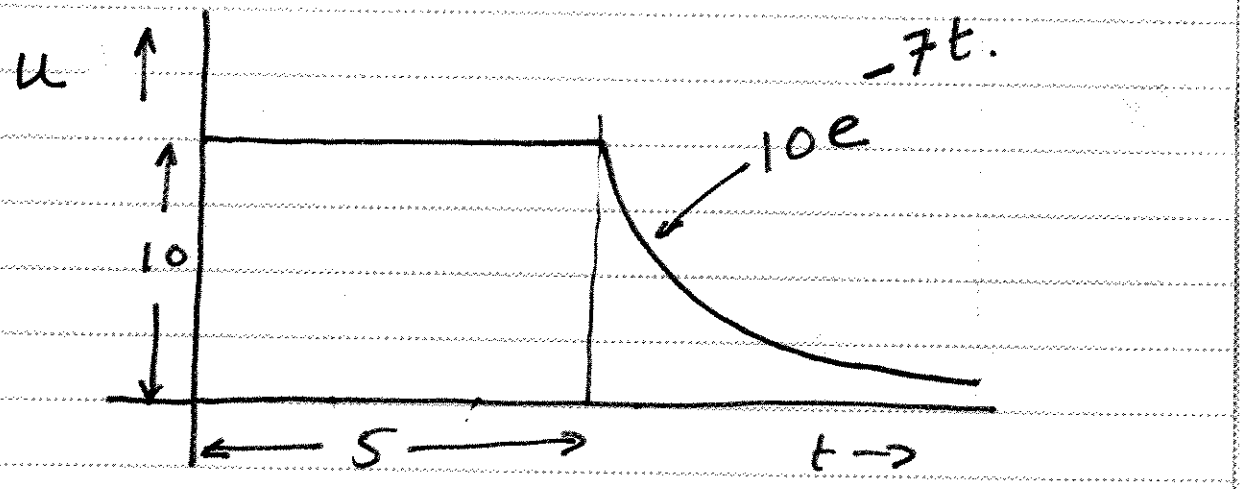
H. W. 8

(1) Consider the following o.d.e.

$$\dot{y}(t) = -2\,y(t) + 5\,u(t)$$

$$y(0) = 0$$

The function $u(t)$ is given as follows



Calculate $y(t)$.

(2) In problem 1 if

$$u(t) = 10 \sin 15t$$

what is $y(t)$?? Also calculate the steady state function $y_{ss}(t)$.

(3) A mass/spring/damper system has eqn

$$\ddot{y} + 2\dot{y} + 4y = u(t)$$

(a) calculate the impulse response of the system (call it $h(t)$).

(b) using convolution, calculate the unit step response.

④ For the following pair of o.d.es.

$$\dddot{y} - 2\dot{y} - 4y = u(t)$$

$$\dddot{y} + 4\ddot{y} + 6\dot{y} + 4y = u(t)$$

do the following.

ⓐ write $x_1 = y$, $x_2 = \dot{y}$, $x_3 = \ddot{y}$ and define $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Write down the equations as

$$\dot{\underline{x}} = A\underline{x} + bu$$

$$y = c\underline{x}$$

Calculate c, A & b.

ⓑ Calculate the characteristic polynomial of A and using matlab calculate the eigenvalues. Which of the

matrices have eigenvalues with negative real parts?

(c) Verify your answer in part (b) using Routh Hurwitz Test. (see Hand out 2)

(d) Calculate the impulse response $h(t)$ for each of the two odes.

$$h(t) = c e^{At} b$$

(e) Choose $u(t) = 1, t \geq 0$ and calculate $y(t)$ using convolution. Can you see which of the two $y(t)$-s remain bounded.

Remark:

The point is to see that there is a connection between boundedness of $y(t)$ and negative realness of the eigenvalues of the matrix A.

(5) Consider the following pair of discrete recursion.

$$y_{k+3} + \frac{3}{2} y_{k+2} + \frac{3}{4} y_{k+1} + \frac{1}{8} y_k = u_k$$

$$y_{k+3} + \frac{9}{2} y_{k+2} + \frac{27}{4} y_{k+1} + \frac{27}{8} y_k = u_k$$

(a) write

$x_1(k) = y(k)$

$x_2(k) = y(k+1)$.

$x_3(k) = y(k+2)$

Define

$$\Sigma_k = \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix}$$

and write

$$\Sigma_{k+1} = A\Sigma_k + b u_k$$

$$y(k) = c\,\Sigma_k.$$

Calculate c, A, b.

(b) Calculate the characteristic polynomial of A and using matlab calculate the eigenvalues. Which of the matrices have eigenvalues with magnitude < 1?

(c) Verify your answer in (b) using Jury's Test. (see Handout 2)

(d) Calculate the impulse response h_k for each of the two recursions.

$$h_k = CA^{k-1}b.$$

(e) Choose $u_k = 1$, $k \geq 0$. and calculate y_k using convolution. Can you see which of the two y_k's remain bounded

Remark: The point is to see that there is a connection between boundedness of y_k and eigenvalues of magnitude < 1.