ESE 501, Home Work 6

1. Assuming that λ_0 is any nonzero real number, compute exponential of the following matrices.

   (a)
$$\begin{pmatrix} \lambda_0 & 1 & 0 & 0 \\ 0 & \lambda_0 & 1 & 0 \\ 0 & 0 & \lambda_0 & 1 \\ 0 & 0 & 0 & \lambda_0 \end{pmatrix} \tag{1}$$

   (b)
$$\begin{pmatrix} \lambda_0 & 0 & 1 & 0 \\ 0 & \lambda_0 & 0 & 1 \\ 0 & 0 & \lambda_0 & 0 \\ 0 & 0 & 0 & \lambda_0 \end{pmatrix} \tag{2}$$

   (c)
$$\begin{pmatrix} \lambda_0 & 0 & 0 & 1 \\ 0 & \lambda_0 & 0 & 0 \\ 0 & 0 & \lambda_0 & 0 \\ 0 & 0 & 0 & \lambda_0 \end{pmatrix} \tag{3}$$

   (d)
$$\begin{pmatrix} \lambda_0 & 0 & 1 & 1 \\ 0 & \lambda_0 & 0 & 1 \\ 0 & 0 & \lambda_0 & 0 \\ 0 & 0 & 0 & \lambda_0 \end{pmatrix} \tag{4}$$

   (e)
$$\begin{pmatrix} \lambda_0 & 1 & 0 & 0 \\ 1 & \lambda_0 & 1 & 0 \\ 0 & 1 & \lambda_0 & 1 \\ 0 & 0 & 1 & \lambda_0 \end{pmatrix} \tag{5}$$

   Remark: I have never tried (d) and (e). You can use any method except typing the matrix into a symbolic computer.

2. (a) Calculate e^A and e^{A^2} assuming A to be the following matrix:
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix} \tag{6}$$

   (b) Verify, using matlab if necessary, that A, e^A and e^{A^2} commute.

   Remark: I am expecting that you will compute the eigenvalues and compute the exponential function by expressing it as a linear combination of I, A and A^2. You can use matlab to compute the linear combination. Do not use the exponential command.

3. (a) Calculate eigenvalues, eigenvectors and generalized eigenvectors of the following matrix.
$$M = \begin{pmatrix} 45 & 120 & 315 & 672 \\ -60 & -252 & -756 & -1680 \\ 45 & 216 & 651 & 1440 \\ -12 & -60 & -180 & -396 \end{pmatrix} \tag{7}$$

   Remark: You can use matlab and if you do, you need to verify your answer by using the definition of eigenvalues, eigenvectors and generalized eigenvectors.

(b) Write e^{Mt} as a linear combination of *I*, *M*, M^2 and M^3 using the eigenvalues of *M*.

(c) Using the matlab 'jordan' command, obtain the matrix *P* such that $P^{-1}MP$ is block diagonal. Verify your results by actually multiplying the matrices using matlab.

4. Show that the matrix

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -24 & 50 & -35 & 10 \end{pmatrix} \tag{8}$$

has an eigenvector at (1¸ λ, λ^2, λ^3) for every eigenvalue λ. Also show that the eigenvalues are at 1, 2, 3 and 4. Write down the matrix *P* (without using matlab) such that $P^{-1}MP$ is diagonal. Now use matlab to verify that $P^{-1}MP$ is indeed diagonal.

5. Manufacture a 10×10 symmetric and positive definite matrix *M*. Calculate eigenvalues of the matrix *M*. Also calculate an orthogonal matrix *P* such that $P^T MP$ is diagonal. Verify by explicit multiplication that $PP^T = P^T P = I$ (You can use matlab for this purpose)

6. Let A be a $n \times n$ matrix. Also assume that the eigenvalues of A are all distinct and let v_i be an eigenvector of A corresponding to an eigenvalue λ_i. Prove that the vectors v_i, $i = 1, \cdots n$ are all linearly independent.