Home Work 4

1 Consider the set S of all vectors in $\mathbb{R}^5$ perpendicular to the following 2 vectors

$$(1 \quad 0 \quad -1 \quad 2 \quad -3)$$

$$(2 \quad 1 \quad 0 \quad 0 \quad -1)$$

You are told that S is a vector space

a. Calculate a basis of S.

b. What is the dimension of S.

c. Use Gram-Schmidt to calculate an orthogonal basis of S.

d. Calculate the projection of the vector

$$(3 \quad 1 \quad -1 \quad 2 \quad -4)$$

on S.

2. Let S be a subspace of $\mathbb{R}^5$ described as follows.

$$S = \{(t_1, t_1+t_2, t_1+t_2+t_3, t_2+t_3, t_3) : t_1, t_2, t_3 \in \mathbb{R}\}.$$

a. Calculate a basis of S and write down its dimension.

b. Calculate the co-ordinates of $(1\ 0\ 0\ 0\ 0)$ w.r.t. the above basis.

c. Orthogonalize the above basis using G.S. algorithm.

d. Is the vector $(1\ 1\ 2\ 1\ 1)$ in S.

e. Project the vector $u = (1\ 1\ 1\ 1\ 1)$ onto S.

f. Calculate the angle between u and $\text{proj}_S u$.

3. (a) Calculate all the 16 minors and hence write down the cofactors and the adjoint matrix of A where

$$A = \begin{pmatrix} 3 & 2 & 1 & -1 \\ 5 & 4 & 5 & 0 \\ 6 & -7 & 6 & -5 \\ 9 & -8 & -12 & 4 \end{pmatrix}$$

(b) Show that

$$A \cdot \operatorname{adj} A = \operatorname{adj} A \cdot A = \begin{pmatrix} \Delta & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & \Delta \end{pmatrix}$$

where $\Delta = \det A$.

(c) Is A invertible? If yes write down

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A.$$

4 (a) Write down the characteristic polynomial of the matrix

$$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$$

(b) Verify by explicit calculations the following (Cayley Hamilton Theorem)

$$A^2 = -3A - 2I$$

(c) Calculate A^{98} without using a calculator.

5. Solve using Cramer's Rule the following set of equations

$$x + 2y + z = 13$$

$$2x - 5y + 3z = 20$$

$$3x - 3y + z = 10$$

6. Write down the characteristic polynomials of the following matrices

(a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ (b) $\left(\begin{array}{cc|cc} 9 & 1 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ \hline 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{array}\right)$

(c) $\begin{pmatrix} 9 & 1 & 0 & 0 \\ 0 & 9 & 1 & 0 \\ 0 & 0 & 9 & 1 \\ 0 & 0 & 0 & 9 \end{pmatrix}$ (d) $\begin{pmatrix} \sigma & \omega \\ -\omega & \sigma \end{pmatrix}$

(e)
$$\left(\begin{array}{cc|cc} \sigma & \omega & 1 & 0 \\ -\omega & \sigma & 0 & 1 \\ \hline 0 & 0 & \sigma & \omega \\ 0 & 0 & -\omega & \sigma \end{array}\right)$$

(f)
$$\begin{pmatrix} 0 & \omega_1 & \omega_2 \\ -\omega_1 & 0 & \omega_3 \\ -\omega_2 & -\omega_3 & 0 \end{pmatrix}$$

(g)
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}$$

(h)
$$\left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -6 & -11 & -6 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -6 & -11 & -6 \end{array}\right)$$

(7)(a) Calculate the rank, nullity, range and null space of the following matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ -1 & 1 & 0 & -3 \end{pmatrix}$$

(b) Find a set of basis vectors for the null space of A.