Home Work 3

Let $V_1 = (1 \quad 3 \quad -1)$

$V_2 = (2 \quad 5 \quad -9)$

be two vectors in $\mathbb{R}^3$.

① calculate $V_1 \times V_2$

②. calculate θ the angle between V_1 and V_2 by using the formula

$$\cos\theta = \frac{V_1 \cdot V_2}{\|V_1\| \, \|V_2\|}$$

Hence calculate $\sin\theta$.

③. Calculate $|V_1 \times V_2|$ and compare your results with 2.

Let $V_3 = (-3 \quad 2 \quad 7)$

④ calculate

$V_1 \times (V_2 \times V_3)$ and $(V_1 \times V_2) \times V_3$

Are they equal??

(5) Calculate

$$|(v_1 \times v_2) \cdot v_3|$$

the magnitude of the scalar triple product of v_1, v_2, v_3.

This number is the volume of a parallelopiped

(6) Show that

$$v_1 \times (v_2 \times v_3) = (v_1 \cdot v_3) v_2 - (v_1 \cdot v_2) v_3 .$$

and

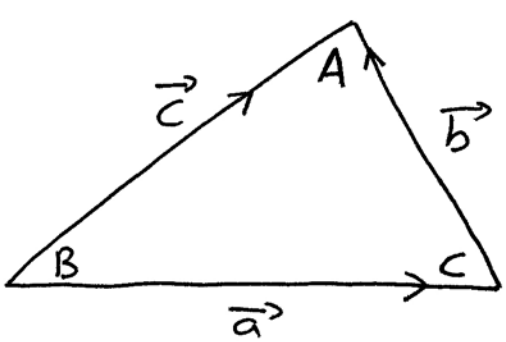
$$(v_1 \times v_2) \times v_3 = (v_1 \cdot v_3) v_2 - (v_2 \cdot v_3) v_1 .$$

This is the vector triple product.

(7) Show that

$$v_1 \times (v_2 \times v_3) + v_2 \times (v_3 \times v_1) + v_3 \times (v_1 \times v_2) = 0$$

(8)



Show that

$$\frac{\|a\|}{\sin A} = \frac{\|b\|}{\sin B} = \frac{\|c\|}{\sin C}$$

Hint: Calculate $c \times a$ and $c \times b$ and $a \times b$ and use your judgement.

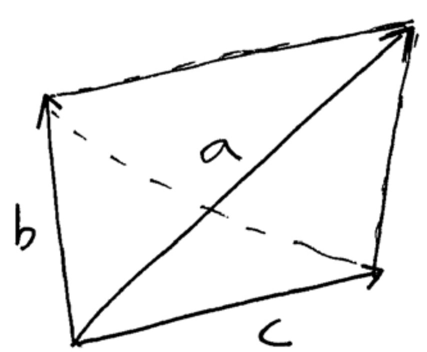
(9) Challenge problem (I have never done this before)

Let v_1, v_2, v_3, v_4 be four vectors in $\mathbb{R}^3$.

Prove the Lagrange Identity

$$(v_1 \times v_2) \cdot (v_3 \times v_4) = (v_1 \cdot v_3)(v_2 \cdot v_4) - (v_1 \cdot v_4)(v_2 \cdot v_3)$$

(10) Less challenging problem



Show that volume of this tetrahedron is

$$V = \frac{1}{6}\left[a \cdot (b \times c)\right]$$

Hint: V is one third of the product of the area of its base and its vertical height.