SSM501, Home Work One

Date of distribution: 08/31/2005

Date Due: 09/07/2005

Please deposit all homeworks in a homework bin allotted for this course in CUPPLES II, Ground Floor (adjacent to the SSM office).

Let us define i to be the vector $(1,0)$ and j to be the vector $(0,1)$.

1. Consider the vector space E^3.

   (a) Calculate the norm of the following vectors

   i.
   $$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$
   ii.
   $$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$
   iii.
   $$\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

   (b) Calculate the angle between every pairs of the above vectors.

   (c) Calculate two vectors of unit length having the same direction as that of the vector
   $$\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

   (d) Using the two vectors
   $$\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$$
   and
   $$\begin{pmatrix} 2 \\ 4 \\ -6 \end{pmatrix}$$
   verify the triangle inequality and Cauchy-Schwarz inequality.

2. Calculate the lengths of each of the following vectors and the angle that each makes with the $x - axis$.

   (a). i+j

   (b) 2i-3j

   (c) $\sqrt{3}$i + j

   (d) 5i+12j

   (e) -5i-12j

3. Find the following vectors.

   (a) A unit vector making an angle of 30° with the positive $x - axis$.

   (b) The unit vector obtained by rotating j through 120° in the clockwise direction.

   (c) A unit vector having the same direction as the vector 3i-4j.

   (d) A unit vector tangent to the curve $y = x^2$ at the point $(2, 4)$.

   (e) A unit vector normal to the curve $y = x^2$ at the point $P(2, 4)$ and pointing from P toward the concave side of the curve.

4. (a) Find the angle between the diagonal of a cube and one of its edges.

   (b) Find the angle between the diagonal of a cube and a diagonal of one of its faces.

   (c) Show that the vector $ai + bj$ is perpendicular to the line $ax + by + c = 0$ in the xy plane.

   The vector that we get by projecting B onto A is called the 'vector projection' of B onto A. We shall denote it by $proj_A B$. Let us define $A = 2i + 5j$ and $B = 3i - 4j$. Calculate the following.

5. (a) $proj_A B$

   (b) $proj_B A$

   (c) C = A - $proj_B A$

   (d) Show that C is orthogonal to B.