H. W 2 (Solutions)

1. If θ is the angle, then

$$\cos\theta = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|} = \frac{107}{10.77 \times 11.18} = .8886$$

$$\theta = .4765 \text{ radians}$$
$$= 27.304^\circ$$

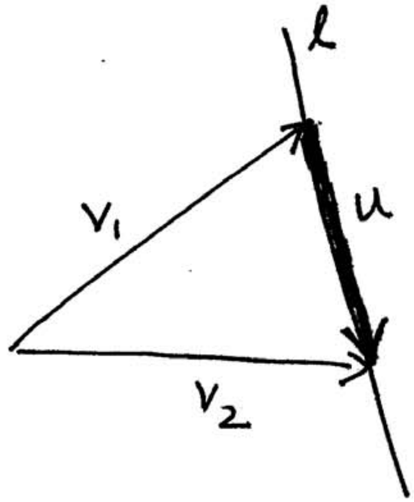
2. $$\text{proj}_{v_1} v_2 = \frac{v_2 \cdot v_1}{v_1 \cdot v_1} v_1$$

$$= \frac{107}{116} \begin{pmatrix} 1 \\ 3 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} .9224 \\ 2.7672 \\ 4.6121 \\ 8.3017 \end{pmatrix}$$

$$\text{proj}_{v_2} v_1 = \frac{v_2 \cdot v_1}{v_2 \cdot v_2} v_2 = \frac{107}{125} \begin{pmatrix} 2 \\ 7 \\ 6 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1.7120 \\ 5.9920 \\ 5.1360 \\ 5.1360 \end{pmatrix}$$

3.



$$u = v_2 - v_1 = \begin{pmatrix} 1 & 4 & 1 & -3 \end{pmatrix}$$

l is parameterized by

$$\{ v_1 + \lambda u : \lambda \in \mathbb{R} \}$$

$$= \{ \begin{pmatrix} 1+\lambda & 3+4\lambda & 5+\lambda & 9-3\lambda \end{pmatrix} : \lambda \in \mathbb{R} \}$$

vector representation

writing

$x = 1 + \lambda$

$y = 3 + 4\lambda$

$z = 5 + \lambda$

$w = 9 - 3\lambda$

we obtain

$$\boxed{x - 1 = \frac{y-3}{4} = z - 5 = \frac{9-w}{3}}$$

Cartesian Equation of a line.

4

$$V_1 = (1 \quad 3 \quad 5 \quad 9)$$

$$e_1 = (1 \quad 0 \quad 0 \quad 0)$$

$$\cos\theta_1 = \frac{V_1 \cdot e_1}{\|V_1\|} = \frac{1}{\sqrt{1+9+25+81}}$$

$$= \frac{1}{10 \cdot 77}$$

Like wise

$$\cos\theta_2 = \frac{3}{10 \cdot 77}$$

$$\cos\theta_3 = \frac{5}{10 \cdot 77}$$

$$\cos\theta_4 = \frac{9}{10 \cdot 77}$$

5. $V_1 \cdot V_2 = 107$

$\|V_1\| = 10{\cdot}77$

$\|V_2\| = 11{\cdot}18$

$\|V_1\| \, \|V_2\| = 10{\cdot}77 \times 11{\cdot}18$

$= 120{\cdot}41$

6. $u_1 = V_2 - V_1 = (1 \quad 4 \quad 1 \quad -3)$

$u_2 = V_3 - V_1 = (-1 \quad 7 \quad -5 \quad 6)$

P is parameterized by

$\{V_1 + \lambda_1 u_1 + \lambda_2 u_2 : \lambda_1, \lambda_2 \in \mathbb{R}\}$

Vector Equation

$$= \left\{ \begin{pmatrix} 1 + \lambda_1 - \lambda_2 \\ 3 + 4\lambda_1 + 7\lambda_2 \\ 5 + \lambda_1 - 5\lambda_2 \\ 9 - 3\lambda_1 + 6\lambda_2 \end{pmatrix} : \lambda_1, \lambda_2 \in \mathbb{R} \right\}$$

7 From 6 we write

$$x = 1 + \lambda_1 - \lambda_2$$

$$y = 3 + 4\lambda_1 + 7\lambda_2$$

$$z = 5 + \lambda_1 - 5\lambda_2$$

$$w = 9 - 3\lambda_1 + 6\lambda_2$$

$$\begin{pmatrix} 1 & -1 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} x-1 \\ z-5 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 7 \\ -3 & 6 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} y-3 \\ w-9 \end{pmatrix}$$

Solving for λ_1 and λ_2 we obtain

$$\lambda_1 = \frac{5x - z}{4} \; ; \; \lambda_2 = \frac{x - z + 4}{4}$$

Substituting λ_1 & λ_2 we obtain.

$$\boxed{27x - 11z - 4y + 40 = 0 ; 9x + 3z + 4w - 60 = 0}$$

Cartesian Equation