

A less known secret about determinants

Let A, B, C, D be four matrices where A is a $m \times m$ matrix, B is a $m \times p$ matrix, C is a $p \times m$ matrix, D is a $p \times p$ matrix. Let us define Q to be the following $(m+p) \times (m+p)$ matrix:

$$Q = \begin{pmatrix} A & B \\ C & D \end{pmatrix}.$$

Fact I:

If A is invertible, i.e. if A^{-1} exists (equivalently $\det A \neq 0$), then

$$\det Q = \det A \det(D - CA^{-1}B).$$

Fact II:

If D is invertible, i.e. if D^{-1} exists (equivalently $\det D \neq 0$), then

$$\det Q = \det D \det(A - BD^{-1}C).$$

Corollary from Fact I:

If $C = 0$, then

$$\det \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} = \det A \det D.$$

Corollary from Fact II:

If $B = 0$, then

$$\det \begin{pmatrix} A & 0 \\ C & D \end{pmatrix} = \det D \det A.$$

Hope this helps in Home Work 5. GOOD LUCK.